1 Introduction

Innovative ideas are financed by independent venture capitalists, angel investors as well as by corporations through corporate venture capital and internal capital markets.[1] A key distinction between financing by independent venture capitalists (hereafter IVC), corporate venture capitalists (hereafter CVC) and angels is the degree of *knowledge sharing* between the innovator and the financier. Given their strategic motivations, CVCs acquire extensive access to their portfolio company's technology. For example, ADE – the corporate VC arm of Analog Devices – usually acquired absolute visitation rights to its portfolio company's R&D facilities and/ or the option to acquire or transfer its technology (Kanter et. al., 1990).[2] In return for such extensive access to the startup's technology, corporate VCs lend their technological expertise to the startups. The Economist (Feb 1999, p. 21) notes that "the high success rate of Xerox Technology Ventures' portfolio companies was due to their accessibility to Xerox's "patent estate" — a concentration of Xerox' proprietary know-how."[3] While IVCs understand the startup's idea, they adopt a more hands-off approach to the startup's *technology* compared to CVCs. Similarly, while IVCs lend their business expertise to the startup,[4] they do not possess a patent portfolio that a corporation can provide access to. Thus, compared to CVC, the give and take of knowledge is lower in IVC. Yet, it is greater than in the passive investments by an angel investor.

Though financiers differ in the degree of knowledge they *share* with the entrepreneur/ innovator, existing literature has not examined its effect on the incentives of the financier and the innovator. Furthermore, existing literature focuses either on the dichotomous choices among CVC, IVC and angel financing or on the choice between financing an idea inside or outside an existing firm. This paper develops a theory of financing choices by *integrating* the incentive effects of *knowledge sharing* in corporate VC, independent VC, and angel financing together with that of financing ideas inside or outside the boundaries of an existing firm. By distinguishing the financing of knowledge assets from that for physical assets, the theory rationalizes why venture capital financing is dominant in the knowledge intensive sectors but not in the traditional sectors. The theory predicts that CVC financing of start-ups is more likely than IVC when Intellectual Property (IP) protection is weaker. This prediction helps explain why (i) start-ups protected by patents are more likely to associate with incumbent corporations than those without a patent (Gans, Hsu and Stern (2002)); and (ii) why research alliances between startups and corporations is widespread in Drugs and Biotechnology (Lerner and Merges (1998)) where patent protection is strong (Cohen, Nelson and Walsh (2000)),

[1] The importance of each of these financing choices can be seen from the following summary statistics. In 2000, corporations invested \$182 billion in internal R&D and \$28 billion through corporate VC. Independent VC financing of startups amounted to \$86 billion in the same year. Chemmanur and Chen (2001) highlight that around 300,000 angel investors invest about \$10-20 billion in close to 30,000 startup firms annually.

[2] More generally, Kann (2000) finds in his analysis of 152 CVC programs that a key goal of CVC programs is to acquire knowledge resources and intellectual property from their ventures. A survey conducted by the US Department of Commerce (MacMillan, et. al. 2008) highlights that CVCs undertake formal agreements for collaboration in the form of R&D or co-development agreements between an operating unit and the portfolio company.

[3] More generally, Stuart (2000) finds in his analysis of over 1600 research alliances that the rate of patenting by startups is positively correlated with the patent portfolio of the corporation funding the research alliance.

[4] A large literature documents the benefits provided by IVCs (see Hellman and Puri, 2002 and references therein).

but not in other industries. The theory also generates new empirical predictions. First, IVC financing of startups is more likely than CVC but less likely than angel financing if IP protection is weaker and if product market competition is more intense. In particular, IP protection becoming stronger affects these financing choices *disproportionately* more when product market competition is intense than when it is moderate. Second, while the choice among IVC, CVC and angel financing is determined by macro-level variables such as IP protection and the intensity of product market competition, the choice between financing an idea inside or outside an existing firm is determined essentially by micro-level variables such as relative abilities of the innovator and the financier. Absent IP protection, however, the choice of financing an idea inside or outside a firm is irrelevant.

The theoretical model builds on the Property Rights Theory of Grossman and Hart (1986), Hart and Moore (1990), Hart (1995) (GHM hereafter). Consider an innovator who finances her idea through a corporation, IVC, or an angel investor. The innovator (financier) owns the IP over the idea if it is financed outside (inside) the boundaries of an existing firm. To capture differences in knowledge sharing, I model *access* between the innovator's and the financier's knowledge assets to be *reciprocally* high in CVC, moderate in IVC, and minimal in angel financing. As in Rajan and Zingales (1998, 2001), I define access as the ability to use, or work with, an asset. Access enables the innovator to familiarize herself to the financier's knowledge and learn to work with it. Crucially, however, access also provides the innovator the opportunity to expropriate the financier's knowledge. Since access is reciprocal, a similar effect applies for the financier too. Therefore, after deciding the financing choice, *both* the innovator and the financier make two kinds of investments — they invest either to tailor their knowledge to that of their partner or to expropriate it.

As in the GHM setup, the innovator and the financier bargain over the total surplus generated by developing the idea. They bargain in the shadow of either a monopoly for the owner of the idea or a duopoly between them. Since knowledge assets are non-rival in nature, potentially both of them can produce on their own after ending their relationship. Whether they both produce or only one of them produces depends upon who owns the idea and the strength of IP protection. If the innovator owns the idea, she enforces her ownership rights by taking the financier to court for infringing her IP rights. If the innovator wins the lawsuit, then the financier is ordered not to produce and the innovator operates a monopoly. If the innovator loses the lawsuit, the financier can produce as well, in which case they compete as a duopoly. Similarly, if the financier owns the idea, a monopoly (duopoly) results if the financier wins (loses) the lawsuit. To account for the *strategic interactions* in a duopoly, I assume that a higher investment by the innovator (financier) not only enhances her (his) profits but also dampens those of financier (innovator); more intense the product market competition, greater the decrease in the financier's (innovator's) profits. If IP protection is stronger, then the owner of the idea is more likely to win the law suit. The innovator's and financier's outside options are the expected value of their monopoly and duopoly profits. In equilibrium, they produce together and operate as a monopoly. Therefore, investment by either agent to tailor his/ her knowledge to that of the partner enhances the pie, i.e. the joint profit. In contrast, investment by either agent to expropriate the other's knowledge only enhances his/ her slice of the pie, i.e., his/ her share of the joint profit. I assume that contracts are *incomplete* and that an increase in access enhances the productivity of either kind of investment.

Thus, the framework employs the following key elements underlying the GHM framework: (i) outside options affect the division of surplus in equilibrium; (ii) outside options are determined by the investments and the boundary/ financing choice; and (iii) due to incompleteness of contracts, all payoffs are unverifiable ex ante but verifiable ex post. The model incorporates three additional features. First, I derive the outside options endogenously by modeling the sub-game that results if outside opportunities are pursued. Second, to capture the efficiency and rent seeking aspects of knowledge sharing, I allow the level of knowledge sharing (and therefore the financing choice) to affect *directly* not only the outside options but also the joint profit. Finally, while in GHM agents choose investments within the relationship (which has spillover effects on their outside options), here, agents' investments affect *directly* the joint profit as well as the outside options.

To start with, I show that while knowledge sharing, i.e. access, is not a choice variable for financing choices involving physical assets, choosing access optimally matters for financing choices involving knowledge assets. Intuitively, the owner of a physical asset can hold-up the user by withdrawing access to the asset ex-post, which leads to under-investment compared to the efficient level. Since access enhances the productivity of investment, full access is *always* optimal in this case. With knowledge assets, however, under-investment in tailoring knowledge assets and over-investment in expropriating them are equally important concerns. Furthermore, access and ownership have contrasting effects on incentives: while ownership *asymmetrically* increases the incentives of the owner and dampens those of the non-owner, access *symmetrically* enhances the incentives of both agents. Therefore, with knowledge assets, choosing *both* ownership and access alleviates simultaneously the twin problems of under- and over-investment by both agents.

Given the contrasting effect of access and ownership on incentives, the trade-offs between the various financing choices are as follows. Since CVC involves more access than IVC, compared to IVC, CVC has the benefit of providing the innovator and the financier stronger incentives to tailor their knowledge to each other but has the attendant cost of providing them both stronger incentives to expropriate each other's knowledge. Since IVC involves greater access than angel financing, the cost and benefit underlying this choice are similar. However, since CVC involves high access while angel financing involves minimal access, both the costs and the benefits mentioned above are more pronounced for this choice. Since the choice between financing an idea inside or outside the boundaries of an existing firm involves a difference in ownership, the trade-off here is as follows. Compared to financing the idea inside, outside financing weakens the financier's incentives both to tailor and expropriate but strengthens the incentives of the innovator to do both.

The theory predicts that the optimal level of access increases as IP protection becomes stronger. In other words, as IP protection becomes stronger, IVC financing of startups becomes less likely than CVC but more likely than angel financing. The intuition for this result is as follows. An increase in IP protection enhances the likelihood of a monopoly for the owner of the idea but decreases that of a duopoly between the innovator and the financier. However, since the innovator's investment has the dual effect of increasing her duopoly profits as well as dampening the financier's duopoly profits, the effect of the rents from a possible duopoly dominates that from a possible monopoly. A similar argument applies for the financier too. Therefore, an increase in IP protection decreases the investment of both agents. Since increasing access enhances both agents' incentives, the optimal level of access increases with an increase in IP protection.

Second, the theory predicts that the optimal level of access decreases if product market competition becomes more intense. An increase in the intensity of product market competition enhances the marginal rents from the innovator's investment by dampening the financier's profits; a similar effect applies for the financier's investment too. Thus, an increase in the intensity of product market competition increases the over-investment in expropriation by both agents. Since decreasing access dampens both agents' incentives, the optimal level of access decreases when product market competition becomes more intense.

Third, an increase in IP protection increases the optimal level of access *disproportionately* more when product market competition is intense than when it is moderate. Since an increase in IP protection decreases the likelihood of a duopoly and the marginal effect of investment is greater when product market competition is intense, the marginal effect of an increase in IP protection on the innovator's and financier's investments is greater when product market competition is intense. The result again follows due to the symmetric effect of access.

Fourth, while the optimal level of access is determined by the strength of IP protection and the intensity of product market competition, optimal ownership of the idea is determined primarily by differences at the firm-level such as the relative abilities of the innovator and the financier and the nature of the knowledge assets. When their knowledge is tacit, neither agent can expropriate easily, which leads both to underinvest compared to the first-best. In this case, if the innovator is relatively more capable than the financier, innovator must optimally own the idea since this alleviates the problem of under-investment better by enhancing the innovator's incentives more than dampening the incentives of the financier. When their knowledge is codified, both agents can expropriate easily, which leads both to overinvest compared to the first-best. Therefore, when knowledge is codified, the result is opposite to that when knowledge is tacit.

Finally, ownership of the idea has no effect on incentives when IP protection is non-existent. This is because once the owner of a knowledge asset provides access, he/ she can enforce her ownership rights only through legal recourse, which is ineffective if IP protection is non-existent.

This paper is organized as follows. The next section reviews the literature. Section 3 describes the model while Sections 4 and 5 analyze the results from the model and detail its empirical implications. While contracts are assumed to be incomplete, Section 6 shows that the results are robust to contracting on cash flow rights and control rights. Further, Section 6 shows that such mechanisms as advocated in Maskin and Tirole (1999) would not alleviate the incentive conflicts that motivate my analysis. Section 7 concludes the paper. Appendix A provides some additional robustness checks while Appendix B contains all the proofs.

2 Review of Literature

This paper is primarily related to the literature on optimal financing choices. Fulghieri and Sevilir (2008) examine how financing innovation inside or outside firms influences an R&D race between two sets of corporations and research units. They also analyze the choice between financing of an independent research unit by CVC vis-a-vis IVC by modeling CVC as an investment by a corporation that consumes the innovation and IVC as an investor that enables the innovation with advice but does not consume it. Aghion and Tirole (1994) analyze how the development of an innovation inside and outside the boundaries of an existing firm affects the efficiency of the innovation. Similarly, Gertner, Scharfstein and Stein (1994) employ ownership to differentiate between financing through internal capital markets vis-a-vis external financing. There are several differences between this study and these prior studies. First, by distinguishing between IVC, CVC and angel financing in the degree of knowledge sharing between the financier and the innovator, we analyze the efficiency and rent-seeking effects of such differing levels of knowledge sharing. This enables us to predict how these financing choices vary with differences in IP protection and the ease of expropriating the knowledge assets. Second, we examine the optimal financing choice among internal capital markets, IVC, CVC and angel financing by analyzing them all *simultaneously*. Fulghieri and Sevilir (2008) analyze separately the choice between financing inside or outside existing firms, on the one hand, and IVC or CVC financing of startups, on the other hand. Aghion and Tirole (1994) and Gertner, Scharfstein and Stein (1994) only focus on financing inside or outside existing firms. Since each of these financing choices — internal capital markets, IVC, CVC and angel financing — is potentially feasible in a particular scenario, our analysis provides a framework to examine the optimal choice among all these alternatives.

Hellman (2002) models CVCs as strategic investors and analyzes when entrepreneurs would choose such strategic investors vis-à-vis purely financial investors such as IVCs. His study predicts that CVC is more likely than IVC when the new idea is complementary to the core business of the corporation. While our theory predicts the same too, it also predicts that *even if complementarity remains unchanged*, CVC is more likely than IVC when IP protection is stronger, product market competition is less intense, or knowledge is less easily expropriable. Bettignies and Chemla (2008) provide a rationale for the choice of CVC versus IVC based on the competition for talented managers.

Chemmanur and Chen (2006) examine the choice between angel financing and IVC financing over the different stages of the idea. In their model, IVCs provide costly effort and add value to their startups while angels do not. In this paper, IVC financing corresponds to greater knowledge sharing (and consequently greater investment by the financier) than angel financing. Mathews (2006) models the use of equity stakes in alliances between an established incumbent and an entrepreneurial firm. In his setting, the transfer of know-how from the entrepreneurial firm to the established firm enhances the efficiency of both firms; however, it also heightens the established firm's incentive to enter the entrepreneur's market. Equity ownership solves the incentive problems by internalizing the effect of the incumbent's entry on the entrepreneur's profits. While this paper resembles

Mathews (2006) in examining efficiency and expropriation together, I examine the incentive effects of reciprocal knowledge sharing (rather than knowledge transfer) as well as differences in ownership in CVC, IVC, angel financing and internal capital markets. Ueda (2004) examines the choice between financing of startups by banks and VCs and predicts that as ideas become easier to steal, an innovator would prefer bank financing to VC financing.

This paper is related to a large literature underlying the theory of the firm. While it combines ownership, as in GHM, and access, as in Rajan and Zingales (1998, 2001), to integrate the primary boundary choices for financing knowledge assets — Independent Venture Capital, Corporate Venture Capital, Angel Financing and Internal Capital Markets — into a unified framework, this paper goes beyond combining the insights in these two sets of theories. First, the paper makes an explicit distinction between the incentive effects of physical and knowledge assets. Second, by endogenously deriving the outside options as a function of ownership of the idea, the theory predicts how product market competition and IP protection affect financing choices.

This paper is part of a large literature on the financing of new ventures through venture capital [see Hellman (1998), Kaplan and Stromberg (2003) and references therein]. These papers, however, focus on the details of the financing arrangements such as the use of convertible preferred stock, the allocation of control rights, and the staging of investments over time. Though I show that my key results are robust to the allocation of cash flow and control rights, such analysis is not the main focus of this study. Instead, in the main model, I abstract from these features and instead examine the incentive issues that arise in the organizational form for financing an idea.

This paper is also related to the literature on innovation and stealing of ideas. Anton and Yao (1994, 2002, 2004) and Baccara and Razin (2006) analyze disclosure of ideas in situations where IP protection is non-existent. We show here that without IP protection, ownership of knowledge assets is economically not important since it does not affect incentives. We proceed to analyze how varying levels of IP protection affects the incentives stemming from ownership and knowledge sharing. Rajan and Zingales (2001) examine how vertical versus horizontal hierarchies can be employed to prevent stealing of ideas by employees. Gromb and Scharfstein (2002) argue that firms can assess their employees better than markets but cannot provide as strong incentives as markets can. Biais and Perotti (2004) show that an unpatentable idea may be safely shared in a partnership with experts who have expertise which complements their idea. Hellman (2006) considers how the trade-off between requiring employees to focus on core tasks versus allowing them to explore new ideas affects the process of idea generation in firms and their development inside/ outside firms. Hellman and Perotti (2006) model ideas as incomplete concepts requiring feedback from agents with complementary abilities for completion and analyze circulation of ideas in firms and in markets.

3 Model

Consider a Scientist (S) who possesses knowledge in developing antibodies for curing diseases. The scientist comes up with a breakthrough idea for an AIDS antibody. The scientist seeks a financier (F) who possesses knowledge assets that are complementary to those of the scientist. To fix ideas, think of these complementary assets as (i) the expertise in technologies that administer such antibodies to humans; (ii) the ability to cost-effectively produce such antibodies through quality and operations management techniques, etc.; and (iii) business expertise in deciding which markets/ customer segments to serve, etc.[5]

Figure 1 summarizes the time line and events in the model. There are three dates, $t = 0, 1$ and 2. To exploit the complementarity in their knowledge assets, at date 0, S and F decide to provide *access* to each other's knowledge. Following Rajan and Zingales (1998, 2001), I define access as the opportunity to use, or work with, an asset. To fix ideas, consider the scientist providing access to her expertise (in antibodies). Access here implies that the financier is exposed to intricate details of the antibody technology by working closely with the scientist. Similarly, the financier providing access (to his drug delivery technology) represents more than just revealing the drug delivery mechanism used for a specific antibody; it implies exposing the scientist to the tacit know-how involved in developing drug delivery mechanisms. As Rajan and Zingales (1998) explain, providing access is not a momentary revealing of information but a process that requires the continued cooperation of the agent providing access.

I model access to be *reciprocal*: S provides as much access to her expertise in developing antibodies as she gets to F's complementary knowledge assets. $\alpha \in [0, 1]$, where 0 implies that access is non-existent while 1 represents full access. Access is full when the scientist and a CVC (as the financier) enter into a joint R&D agreement in which the employees in their respective R&D departments meet periodically and share their expertise. At the other extreme, if a passive angel investor finances the scientist's idea, then access is non-existent. In between these two extremes, if an IVC finances the idea, then the scientist reveals her idea to the IVC and exposes the IVC to all such details as are required to assess the economic feasibility of the idea. In return, the IVC provides access to his business expertise. However, the scientist and the financier do not contract to undertake joint R&D; neither does the IVC contract for absolute visitation rights to the scientist's R&D facilities (as in the example of ADE provided in the Introduction). On the flip side, the IVC does not possess patent portfolios or the know-how that provides a corporation its competitive advantage. Thus, access being reciprocally high, moderate and low in CVC, IVC and angel financing captures the level of knowledge sharing between the scientist and these categories of financiers.

Apart from deciding access at date 0, S and F decide the ownership of the idea. Let $\delta = 0$ denote that S owns the idea while $\delta = 1$ denote that F owns it. As I explain in Section 3.2.1 shortly, $\delta = 0$ and $\delta = 1$ imply that S and F respectively have the right to seek legal recourse if their intellectual property rights (IPR) are infringed upon.

[5] It may be useful to comment briefly on the motivation for focussing on these assets. Note that a Corporation does possesses other physical assets which the scientist may benefit from – manufacturing facilities, sales and distribution network, brand name, etc. Similarly, a Venture Capitalist's reputational capital is very valuable to the scientist in developing the idea. However, unlike the knowledge assets that we are focusing on, these assets cannot be expropriated by the scientist. Since our focus on the efficiency and rent-seeking effects of knowledge sharing between the financier and the scientist, we ignore these assets.

We assume that both the scientist and the financier are risk-neutral; furthermore, the financier is not liquidity constrained but the scientist is.

3.1 Investment

As in Rajan and Zingales (1998, 2001), access enables users to *specialize* to an asset and learn how to work with it. For example, F learns about the antibody technology so that he can tailor his complementary assets to the same while S learns how to use the complementary assets so that she can tailor the antibody to the same. However, as in Rajan and Zingales (2001), access also provides each user the opportunity to possibly *expropriate* the knowledge asset of their partner. Thus, on getting access, F may try to expropriate S's antibody technology while S may try to expropriate F's complementary assets. In order to expropriate the antibody technology, F and S have to understand it in its *totality*. In contrast, to specialize/ tailor her antibody technology to F's complementary asset, S needs to understand *only* those features that affect how well the antibody technology fits together with the complementary asset. Similarly, to specialize, the financier needs to understand only those features that affect how well the complementary asset fits together with the antibody technology.

The scientist and the financier make investments e^S and e^F respectively at date 1. F makes investments (a) to tailor his complementary asset to S's antibody, and (b) to understand and possibly expropriate S's antibody technology (similar for S). As Dushnitsky and Shaver (2007) argue, knowledge sharing can be costly to innovators since the corporation can imitate the innovation. On the flip side, innovators can potentially expropriate the knowledge assets of the financier too.[6]

To economize on the notation, I do not model explicitly the multi-tasking in investments here though I show in Appendix A.1 that the results are identical when multi-tasking is incorporated into the model. The cost of investment is equal to the level of investment. As in the Property Rights framework, I assume that these investments are *observable but not verifiable.*

3.2 Joint Profit and Outside Options

At date 2, the scientist and the financier decide whether they would produce together or separately. Denote the joint profit, i.e., the expected profit if S and F produce together, by R. Denote the scientist's and the financier's outside options, i.e. the expected profits if they were to produce individually, by r^S and r^F. If S and F decide to continue their relationship at date 2, they bargain over the split of the joint profit R using 50 : 50 Nash bargaining. As shown in Appendix A, the results are robust to alternative bargaining solutions.

Now, I derive the outside options endogenously by modeling the sub-game that results when S and F decide to go their separate ways at date 2. Before describing this sub-game, it is critical to highlight some important differences between knowledge assets and physical assets.

[6] Bhide (2000), for example, finds that 71% of the firms in the Inc 500 were founded based on ideas encountered in previous employment.

3.2.1 Control over physical assets versus control over knowledge assets

To draw this distinction, recall Coase (1937)'s argument that, in contrast to arms-length transactions, transactions inside firms are organized through power or fiat. In this framework, GHM propose that ownership of (nonhuman) assets provides power since the owner can hold up the user at date 2 after the user has specialized to the asset at date 1. Crucially, the owner can hold up the user only because the *owner can withdraw access to the physical asset at date* 2.

We define knowledge assets as those that are *non-rival in nature*. In other words, in contrast to physical assets, the owner of a knowledge asset cannot withdraw access ex-post after she has provided the same ex-ante. Say that S owns the idea. Then, in our setting, though F does not have expertise over S's antibody technology at date 0, F can acquire the same at date 2. Since S cannot withdraw access to her antibody technology ex-post, F can utilize it for production along with its complementary assets.[7] In this scenario, S can enforce its ownership rights only through legal recourse, i.e. sue F for infringing on its Intellectual Property Rights (IPR) and seek damages (Crampes and Langinier, 2002). Thus the owner of a knowledge asset can only hold up the user by seeking legal recourse (or by threatening to do so).[8] This is in contrast to a physical asset, where the owner can hold up by explicitly withdrawing access to the asset. I model this feature of knowledge assets in the sub-game that follows if S and F decide to go their separate ways.

3.2.2 Product Market Characteristics and Profits

I assume that if S and F produce together, they have a monopoly over the product that they produce. Similarly, if $S(F)$ can produce but $F(S)$ cannot, then $S(F)$ operates as a monopoly. If S and F decide to produce together, their joint profit R is a function of the investments that S and F make to understand each other's knowledge assets and their investment to tailor them to each other. Therefore,

$$R = a + \beta \cdot \phi\left(e^S, \alpha\right) + \beta \cdot \phi\left(e^F, \alpha\right) \tag{1}$$

[7] Note that even if S's knowledge asset is patented, still F may be able to get away with utilizing it for production. As Cohen, Nelson and Walsh (2000) document in their survey evidence on intellectual property protection for over 140 4-digit SIC industries, protection accorded to product and process patents are an exception rather than a norm: only in a few industries such as Chemicals and Drugs and Biotechnology, patents enable the owner to protect their IP assets. In most other industries, patenting is ineffective in protecting IP assets; firms rely on secrecy and other informal mechanisms to extract rents from ownership of IP.

[8] The right stemming from ownership of knowledge assets can be illustrated through several anecdotes of litigation and/ or renegotiation following infringement of intellectual property. The most famous example is the 1980's patent suit between Kodak and Polaroid, in which Polaroid was awarded over $1 billion in damages for alleged patent infringement by Kodak on Polaroid's Instant Camera. Kodak was ordered to stop production of its own version of the instant camera and was forced to withdraw from this market. In this example, though Kodak gained access to Polaroid's knowledge through its patents, Polaroid was able to impose its ownership rights by taking Kodak to court. The recent patent infringement suit filed by Verizon against Vonage is similar to the Polaroid-Kodak case. Also in the 1980s, Texas Instruments sued several Japanese and Korean semi-conductor manufacturers for infringing on its patents on the DRAM technology. It simultaneously initiated negotiations about the royalty payments on these patents and was able to increase its royalty payments by ten times (!) in return for withdrawing these patent infringement suits. In this example, Texas Instruments gave access through licensing agreements and imposed its legal rights from ownership by renegotiating under the threat of litigation.

where $a > 0$ is a constant, and $\beta > 1$ captures the degree of complementarity that stems from pooling the scientist's knowledge with that of the financier. To allow for the interesting scenario where both under- and over-investment are possible, I assume that

$$1 < \beta < 2 \tag{2}$$

If S operates as a monopoly, then the profit that S can generate is a function of the investment that S makes to understand and possibly expropriate F's asset. Therefore the profits that S generates as a monopoly $\pi^{S,M}$ is given by

$$\pi^{S,M} = a + \theta^S \phi\left(e^S, \alpha\right) \tag{3}$$

Note that since knowledge assets are non-rival in nature, S can utilize the expertise that she acquires from F about the complementary asset. However, she may not be able to acquire all of F's knowledge. Therefore, I assume that the profits that S can generate are lower than that obtained when S and F produce together. Thus, compared to the expression in (1), the profit function above does not contain $\beta > 1$. $\theta^S \in [0, 1]$ denotes how easily S can expropriate F's knowledge asset. θ^S is a function of (i) S's intrinsic ability; and (ii) the technological characteristics of F's complementary asset. If it is tacit and complex, then the scientist would find it difficult to expropriate; in contrast, if it consists of simple, codified knowledge, then it is easy to expropriate.

Similar to (3), the profit produced by F when operating as a monopoly is given by

$$\pi^{F,M} = a + \theta^F \phi\left(e^F, \alpha\right) \tag{4}$$

where $\theta^F \in [0, 1]$ denotes how easily F can expropriate S's antibody technology.

If *both* S and F can produce separately, then they compete with each other in a duopoly with *differentiated products*. Since both S and F acquire each other's knowledge after receiving access, they will compete with each other if they can both produce. However, the financier (scientist) may not be able to learn all that the scientist (financier) knows about the antibody technology (complementary asset). Therefore, their products will be differentiated from each other. We allow for strategic interactions between S and F when they compete in a duopoly. We assume that in this case their profits, denoted by $\pi^{S,D}$ and $\pi^{S,D}$, are given by

$$\begin{aligned} \pi^{S,D} &= 0.5a + \theta^S \phi\left(e^S, \alpha\right) - \gamma \theta^F \phi\left(e^F, \alpha\right) \\ \pi^{F,D} &= 0.5a + \theta^F \phi\left(e^F, \alpha\right) - \gamma \theta^S \phi\left(e^S, \alpha\right) \end{aligned} \tag{5}$$

where $0 < \gamma < 1$ captures the degree to which F's product features dampen S's profits and vice-versa; thus, γ captures the intensity of product market competition S and F face if they produce independently; $\theta^S, \theta^F \in [0, 1]$ denote how easy S and F find expropriating their partner's technology.[9] The above modeling of profits for the case of a duopoly is intended to capture the

[9] This profit function will be obtained for example if S and F compete (Cournot) Bertrand and face linear (inverse)

following feature of product market competition. If S can expropriate F's complementary asset easily and/ or if S's investment to expropriate is greater, then S can combine better the expertise that she acquires over F's complementary asset with her original expertise in antibodies. This enables S to not only generate greater profits for herself but also dampen F's profits. A similar effect applies for F too.

3.2.3 Ownership, IP Protection and Outside Options

To examine the effect of ownership and Intellectual Property (IP) protection on the outside options, consider first the case where S owns the idea, i.e. $\delta = 0$. Therefore, S has the right to sue F while F cannot do so. We assume that with probability μ, S wins the IP infringement suit and loses with a probability $(1-\mu)$; thus, *μ captures the strength of IP protection.* If the idea is patented at date 2, then μ reflects the strength of patent protection. As Cohen, Nelson and Walsh (2000) find in the Carnegie Mellon University survey, strong patent protection is an exception than a norm. Therefore, even if the idea is patented at date 2, the owner may not be able to enforce ownership rights with certainty. Note that what matters is whether the idea is patented or not at date 2. Therefore, ideas that are not patented at date 0 but receive a patent by date 2 fall into the same category as ideas that are patented at date 0. If the idea is unpatentable or is not patented at date 2, then effectively $\mu = 0$.

If S wins the infringement suit against F, then F is ordered out of the market and therefore cannot produce. In this case, S operates as a monopoly. If S loses, then both S and F can produce and they compete as a duopoly. Thus, S's and F's outside options when S owns the idea, denoted by $r^S(\delta = 0)$ and $r^F(\delta = 0)$ respectively, are given by

$$\begin{aligned} r^S(\delta = 0) &= \underbrace{\mu \cdot \pi^{S,M}}_{\text{Monopoly for S}} + \underbrace{(1-\mu) \cdot \pi^{S,D}}_{\text{Duopoly}} \\ r^F(\delta = 0) &= \underbrace{\mu \cdot 0}_{\text{Monopoly for S}} + \underbrace{(1-\mu) \cdot \pi^{S,D}}_{\text{Duopoly}} \end{aligned} \tag{6}$$

where $\pi^{S,M}$ is given by (3) and $\pi^{S,D}$ and $\pi^{F,D}$ are given by equation (5).

Using a similar reasoning, it follows that S's and F's outside options when F owns the idea, denoted by $r^S(\delta = 1)$ and $r^F(\delta = 1)$ respectively, are given by

$$\begin{aligned} r^S(\delta = 1) &= \underbrace{\mu \cdot 0}_{\text{Monopoly for F}} + \underbrace{(1-\mu) \cdot \pi^{S,D}}_{\text{Duopoly}} \\ r^F(\delta = 1) &= \underbrace{\mu \cdot \pi^{F,M}}_{\text{Monopoly for F}} + \underbrace{(1-\mu) \cdot \pi^{F,D}}_{\text{Duopoly}} \end{aligned} \tag{7}$$

---

demand functions, where the (quantity demanded) price that they charge decreases with not just their (own price) quantity that also that of the other firm.

Using

$$r^i\left(.,\delta\right)=\delta r^i\left(\delta=1\right)+\left(1-\delta\right)r^i\left(\delta=0\right),i=S,F$$

we get

$$r^S\left(e^S,e^F,\alpha,\delta\right)=a\left(0.5+0.5\mu-\delta\mu\right)+\underbrace{\left(1-\delta\mu\right)\theta^S\phi\left(e^S,\alpha\right)}_{\text{Effect of own investment}}-\underbrace{\gamma\left(1-\mu\right)\theta^F\phi\left(e^F,\alpha\right)}_{\text{Effect of partner's investment}} \tag{8}$$

$$r^F\left(e^S,e^F,\alpha,\delta\right)=a\left(0.5-0.5\mu+\delta\mu\right)+\underbrace{\left(1-\mu+\delta\mu\right)\theta^F\phi\left(e^F,\alpha\right)}_{\text{Effect of own investment}}-\underbrace{\gamma\left(1-\mu\right)\theta^S\phi\left(e^S,\alpha\right)}_{\text{Effect of partner's investment}}$$

The scientist's outside option, which equals the expected value of the profits from the monopoly and duopoly scenarios, increases with her own investment in expropriating F's complementary assets but decreases with F's investment in expropriating her antibody technology; the decrease is disproportionately higher when product market competition is more intense (similar for F).

3.2.4 Outside Options for Physical Assets

To compare financing of knowledge assets vis-a-vis that of physical assets, let us evaluate the outside options for S and F if their financing relationship involved physical assets. In other words, S seeks financing for an idea involving a *physical asset* and F owns a complementary physical asset which it lets S utilize for production. Since access provided to physical assets can be withdrawn by the owner, the user of the asset is deprived of its use if the scientist and the financier decide not to collaborate with each other at date 2.

As in Gertner, Scharfstein and Stein (1994), we assume that if F(S) owns the IP over the idea, then F(S) owns the physical asset that is financed as well. Consider first the case where F owns the idea. Therefore, F owns both the assets while S owns neither. Therefore, F can utilize the expertise acquired over S's asset along with his expertise in the complementary asset. However, since S owns neither asset and F can deprive her of access to both assets, S cannot produce outside their relationship. Thus, given the product market characteristics described above, F operates as a monopoly. Thus, the outside options, denoted by $r^S\left(\delta=1,Physical\right)$ and $r^F\left(\delta=1,Physical\right)$, are given by

$$\begin{aligned}r^S\left(.,\delta=1,Physical\right)&=0\\r^F\left(.,\delta=1,Physical\right)&=a+\theta^F\phi\left(e^F,\alpha\right)\end{aligned}$$

where $\theta^F\in[0,1]$ captures how easily F can expropriate S's expertise over her asset.

Now consider the case where S owns the idea and therefore the financed asset. In this case, if S and F discontinue their relationship, both of them can withdraw access to their respective assets. Since both can produce, albeit with the use of *only one asset*, a duopoly results. Therefore, their outside options, denoted by $r^S(\delta = 0, Physical)$ and $r^F(\delta = 0, Physical)$, are given by

$$\begin{aligned} r^S(., \delta = 0, Physical) &= 0.5a + \beta_P\left[\theta^S\phi\left(e^S, \alpha\right) - \gamma\theta^F\phi\left(e^F, \alpha\right)\right] \\ r^F(., \delta = 0, Physical) &= 0.5a + \beta_P\left[\theta^F\phi\left(e^F, \alpha\right) - \gamma\theta^S\phi\left(e^S, \alpha\right)\right] \end{aligned} \tag{9}$$

where $0 < \beta_P < 1$.

Note that with physical assets, any expertise gained from specializing to the asset is useless without having access to the asset itself. This is in contrast to knowledge assets where the expertise gained over the asset cannot be separated from the asset itself. In other words, with knowledge assets, the scientist and the financier could utilize for production the expertise gained over their partner's (knowledge) asset unless legally restricted by the court, which is not possible with physical assets. Therefore, comparing with (5), we can see that $0 < \beta_P < 1$ captures the reduction in profits from producing without the partner's asset. To focus on the case where the complementarity stemming from using both assets together is significant enough, we make the following parametric assumption

$$\beta_P < \beta - 1 \tag{10}$$

Using the above expressions, the outside options for physical assets can be expressed as[10]

$$\begin{aligned} r^S\left(e^S, e^F, \alpha, \delta, Physical\right) &= 0.5a(1-\delta) + (1-\delta)\beta_P\left[\theta^S\phi\left(e^S, \alpha\right) - \gamma\theta^F\phi\left(e^F, \alpha\right)\right] \\ r^F\left(e^S, e^F, \alpha, \delta, Physical\right) &= 0.5a(1+\delta) + \left[(1-\delta)\beta_P + \delta\right]\theta^F\phi\left(e^F, \alpha\right) - \gamma(1-\delta)\theta^S\phi\left(e^S, \alpha\right) \end{aligned} \tag{11}$$

3.3 Technology

I assume that the function ϕ is increasing and concave in its arguments, i.e.

$$\phi_e > 0, \phi_{ee} < 0, \phi_\alpha > 0, \phi_{\alpha\alpha} < 0, \phi_{ee}\phi_{\alpha\alpha} > \phi_{e\alpha}^2 \tag{12}$$

As in Rajan and Zingales (1998, 2001), I assume that if access provided is higher, agents are exposed to more intricate details and can learn and familiarize themselves to the same. Therefore, increasing access makes specialization as well as expropriation more efficient. Therefore, I assume

$$\phi_{e\alpha} > 0 \tag{13}$$

3.4 Nature of contracts

As in PRT, contracts are assumed to be incomplete. Explicitly, two important assumptions characterize the incomplete contracts environment. First, the date 1 investments are observable but not verifiable. The inability to contract on investment can be motivated by the fact that intra-

[10] The outside options for physical and knowledge assets can be embedded into one general functional form by letting $\mu = 1$ for physical assets and incorporating β_P for physical assets in the profit function for duopoly (5). However, for exposition purposes, we have dealt it separate.

firm activities such as the nature or extent of its investment are difficult to verify. This is especially true in our setting involving knowledge sharing and innovation since the tailoring and expropriating investments are quite intertwined with each other. Second, the payoffs R, r^S and r^F are assumed to be non-contractible ex-ante, i.e. at date 0, though they are contractible ex-post, i.e. at date 2.

As Tirole (1999) explains, these assumptions can be justified by invoking two more primitive assumptions. First, the contract at date 0 cannot specify in detail all the different contingencies that may arise — a situation that Tirole (1999) labels "indescribable contingencies." However, indescribability would not limit the menu of contracts that can be written at date 0 if the two parties can commit to a contract that would not be renegotiated at date 2. Indescribability leads contracts to be incomplete when renegotiation is possible "because the constraints imposed by renegotiation make it harder to make up for the information garbling that is implied by the indescribability of contingencies." (Tirole, 1999, pp. 761).[11] The assumption of indescribable contingencies is natural to the setting being studied here because innovation involves considerable exploration (see Manso, 2008).[12] Given such uncertainties, it is unlikely that the two firms will be able to anticipate all possible contingencies and contract upon the specific details of the activities, investment, etc. entailed in developing a product using their respective technologies. Furthermore, the significant exploration uncertainties inherent to innovative activities leaves room for ex-post negotiation.

Since we model the outside options endogenously, the assumption that r^S and r^F are non-contractible ex-ante requires that S and F cannot contractually agree at date 0 not to compete. This assumption is natural given the fact that explicit agreements in restraint of trade are illegal in the United States under Section 1 of the Sherman Act.

Kaplan and Stromberg (2003) document that VCs write detailed contracts specifying the allocation of cash flows and control rights. Therefore, given our analysis of financing choices, we show in an extension of the model in Section 6.1 that our results are robust even if we allow the scientist and the financier to contract on the division of cash flows and the allocation of control.

Maskin and Tirole (1999), in their critique of incomplete contracts, argue that the inefficiency of under-investment stemming from incompleteness in contracts can be rectified through a "mechanism", which involves joint ownership of the asset together with options to sell. We also show in our setting that such a mechanism cannot solve the incentive distortions that motivate our analysis.

3.5 Solving the model

The model is solved using backward induction. The following Lemma leads to the conclusion that the outside options are never exercised in equilibrium.

[11] Note that, given indescribability and renegotiation, revenue-sharing rules contracted at date 0, incentive contracts, contracts that explicitly specify performance at date 2, or mechanisms that involve messaging between the two parties or to third parties become impotent in solving the incentive problem that is analyzed in this paper [see Hart (Chapter 4, 1995) for details].

[12] Though uncertainty is not modeled explicitly, the following analysis is not affected if we replace the payoffs at date 2 by their expectations [see Hart and Moore(footnote 5, 1990)].

Lemma 1:

$$r^S + r^F < R \; \forall e^S, e^F, \theta^S, \theta^F, \alpha, \delta$$
$$r^S\,(Physical) + r^F\,(Physical) < R \; \forall e^S, e^F, \theta^S, \theta^F, \alpha, \delta$$

Thus, neither the financier nor the scientist exit the relationship to develop the idea on their own. Therefore, the outside options only affect the division of surplus in equilibrium.

The total surplus is given by

$$TS = R - e^S - e^F \tag{14}$$

Given the assumption of 50:50 Nash bargaining at date 2, each agent's share of the surplus is given by

$$\begin{aligned} \Pi^S &= 0.5\left(R + r^S - r^F\right) - e^S \\ \Pi^F &= 0.5\left(R - r^S + r^F\right) - e^F \end{aligned} \tag{15}$$

The investments are chosen at date 1. The first-best investments $\left(e^{FF}, e^{SF}\right)$, which maximize TS, are given by the following first order condition:

$$R_i\left(e^{iF}\right) = 1, i \in \{S, F\} \tag{16}$$

The second-best Nash equilibrium level of investments $\left(e^{F*}, e^{S*}\right)$, which firm i chooses to maximize Π^i, are given by

$$\begin{aligned} R_S\left(e^{S*}\right) + r_S^S\left(e^{S*}\right) - r_S^F\left(e^{S*}\right) = 2 \\ R_F\left(e^{F*}\right) + r_F^F\left(e^{F*}\right) - r_F^S\left(e^{F*}\right) = 2 \end{aligned} \tag{17}$$

Since $\phi_{ee} < 0$, $\left(e^{SF}, e^{FF}\right)$ and $\left(e^{S*}, e^{F*}\right)$ exist and are unique.

Given the equilibrium level of investments, define the equilibrium net surplus as

$$TS^* = R\left(e^{S*}, e^{F*}, \alpha\right) - e^{S*}(\alpha, \delta) - e^{F*}(\alpha, \delta) \tag{18}$$

Since the scientist originates the idea, she owns the IPR to the idea to start with.[13] Therefore, we assume the scientist has all the bargaining power at date 0. Since the financier is not financially constrained, ex-ante private transfers from the financier to the scientist are possible. Therefore,

[13] Even if the innovator is an employee of a corporation and got the idea while working for the firm, this would still be true. Gilson (1999) notes that: "Who owns an invention discovered by an employee depends on the stage of the inventive process at which the question is asked. The critical point in the process is "conception," defined as "the first occurrence of the complete invention in the mind of the inventor - as corroborated by objective evidence." Under the law of inventions, ideas remain the employee's property until conception. And because conception requires the employee to take the affirmative step of creating written corroboration, an employee can choose to delay this event until after he leaves the company."

(α^*, δ^*) can be chosen to maximize the joint surplus.

$$(\alpha^*, \delta^*) \equiv \underset{(\alpha,\delta)}{\arg\max} \;\; TS^* \tag{19}$$

3.6 Modeling Financing Choices using Access and Ownership

Before we describe the results of the model, let us examine how access and ownership map into the various financing choices.

As in Aghion and Tirole (1994) and Gertner, Scharfstein and Stein (1994), ownership of the idea provides an essential distinction between an idea being financed inside the boundaries of a corporation, i.e., internal capital markets,[14] and the idea being financed outside its boundaries, i.e. through a startup. Specifically, the scientist (financier) owns the idea when it is financed through a startup (internal capital markets). Within internal capital markets, the reciprocal access between the corporate headquarters, which is the financier, and the acquired division may be high, moderate or low. This captures the level of reciprocal knowledge sharing that corporate headquarters chooses to set between the acquired division and the rest of the firm.

When a startup is created, either CVC, IVC, or angel financing are possible.[15] I now explain why I model knowledge sharing in CVC, IVC and angel financing to be respectively high, moderate and low.

3.6.1 Corporate VC, Independent VC and Angel Financing

When a corporation invests in a startup, there is a *quid pro quo* arrangement between the innovator and the corporation: In return for letting the innovator use its technological expertise, the corporation seeks to know the details of the innovator's technology. The Economist (Feb 1999, p. 21) suggests that the high success rate of Xerox Technology Ventures' portfolio companies was due to their proximity to an unusually high "patent estate" – a concentration of Xerox' proprietary know-how. Similarly, Intel Capital's portfolio companies got the benefit of working with Intel labs on cutting-edge technology issues (Kanter et. al., 1990). To illustrate this benefit using a specific example, examine this statement by the founder of Bipolar Integrated Technology, a portfolio company of Analog Devices' VC arm (Source: Kanter et. al., 1990):

> "Ours was such an advanced technology that it was unrealistic to go to venture capitalists and expect them to understand it. We were going to *need a company within the*

[14] A corporation financing the idea through internal capital markets can occur in two ways: either an employee of the corporation can come up with the idea and continue to work for the corporation or an independent university researcher can sell her idea to the corporation and become its employee. In either case, the innovator relinquishes the right to decide how to develop the idea apart from transferring the legal ownership to innovations generated from the idea. Therefore, the corporation owns the idea when it is developed inside its boundaries.

[15] As argued in footnote 13, the scientist owns the IP to the idea. Therefore, even if the innovator is an employee of a corporation and got the idea while working for the firm, each of these financing choices is feasible. Gilson (1999) notes that the enforcement of the non-compete clauses in employment agreements has been difficult, particularly in California. Consistent with this, Bhide (2000) finds that 71% of the firms in the Inc 500 were founded based on ideas encountered in previous employment.

industry that would understand and support our technology." (Emphasis added)

In return for providing access to their technological expertise, corporations acquire one or more of the following control rights in their portfolio companies: a window into the new technology through absolute visitation rights to the company's R&D facilities, the option to acquire the technology, and technology transfer agreements (Kanter et. al., 1990). In sum, access between the financier and the scientist is reciprocally high when a corporation finances the startup through either CVC investments or through research alliances.

Compared to CVC, IVC financing involves lower knowledge sharing. As Gompers and Lerner (1998) point out, while CVCs hope to get a window into the innovator's technology, IVCs invest strictly for financial gains. As a result, control rights such as absolute visitation rights to the company's R&D facilities, the option to acquire the technology, or technology transfer agreements do not feature in VC contracts (see Kaplan and Stromberg, 2003 for a description of the control rights in VC contracts). Furthermore, compared to CVCs, IVCs adopt a more hands-off approach with respect to the technology of the company. Instead, they lend their business expertise and concentrate on the business aspects such as team building, setting up human resource management policies, recruiting executives, etc. (Hellman and Puri, 2002). Therefore, owing to differences in their investment motivations, the level of knowledge sharing is lower in the case of IVC financing.

Yet, compared to an angels, IVCs try to understand the startup's idea and lends their business expertise to the startup. A large literature documents the advice and expertise that IVCs provide to startups (see Hellman and Puri (2002) and references therein). Therefore, we model access to be reciprocally high in CVC, moderate in IVC, and low in angel financing.

4 Results

Examining the expressions (8) for r^S and r^F leads to our first result:

Proposition 1 (Ownership enhances total and marginal value of outside options):
(a) $r^S(\delta=0) > r^S(\delta=1)\,;\; r^F(\delta=0) < r^F(\delta=1)$
(b) $r^S_S(\delta=0) > r^S_S(\delta=1)\,;\; r^F_F(\delta=0) < r^F_F(\delta=1)$

Parts (a) and (b) of the Proposition state that ownership of the idea enhances respectively the total and marginal values of the outside options. Thus, the outside options derived here exhibit properties that have been employed in the existing literature. In Aghion and Tirole (1994), ownership of an idea enhances the total surplus while in Hart and Moore (1990) and Hart (1995), ownership enhances the marginal value of outside options.

Proposition 2 (Marginal Value from Ownership increases with IP Protection):
(i) If $\mu = 0$, then for $i = S, F$, then $r^i(\delta=0) = r^i(\delta=1)\ \ \forall \alpha, e^S, e^F, \gamma, \beta, \theta^S, \theta^F$.
(ii) $\frac{d\left[r^S_S(\delta=0) - r^S_S(\delta=1)\right]}{d\mu} > 0\ \forall \alpha, e^S, \gamma, \beta, \theta^S$; $\frac{d\left[r^F_F(\delta=1) - r^F_F(\delta=0)\right]}{d\mu} > 0\ \ \forall \alpha, e^F, \gamma, \beta, \theta^F$.

Part (i) of the above Proposition states that ownership of knowledge assets has no effect on the outside options if IP protection is non-existent. Part (ii) of the Proposition states that ownership *disproportionately* enhances the marginal value of outside options when IP protection becomes stronger. As we argued in Section 3.2.1, since knowledge assets are non-rival, an owner can enforce his/ her IPR only through legal recourse. Therefore, if legal protection provided to IP is non-existent, then the right stemming from ownership of IP is economically meaningless. Thus, ownership of IP matters for incentive purposes only in the presence of legal enforcement of IP rights. Furthermore, since an owner can enforce his/ her IPR only through legal recourse, as IP protection becomes stronger, the marginal value of ownership on outside options increases disproportionately.

Proposition 3 (Under- and over-investment): Given α, δ, for $i = S, F$, (i) $\Pi_i^i \leq TS_i \ \forall e^i \Rightarrow e^{i*} \leq e^{iF}$, (ii) $\Pi_i^i > TS_i \ \forall e^i \Rightarrow e^{i*} > e^{iF}$.

S and F underinvest (overinvest) when their investment enhances the marginal value of joint profit more (less) than the marginal value of their share of the joint profit. The intuition for this result is quite standard: the first-best level of investment maximizes the joint profit net of the cost of S's and F's investments while the second-best investment results when each agent maximizes her share of the joint profit net of the cost of her investment. This result is similar to that in Hart (1995) – the only difference is that condition (b) is not considered in their analysis. Baker, Gibbons and Murphy (2002) examine overinvestment and underinvestment compared to the first-best benchmark and analyze how relational contracts can address these incentive distortions.

4.1 Benchmark Case of Physical Assets

As we argued in Section 3.2.1, the essential difference between physical assets and knowledge assets is that access can be withdrawn ex-post to physical assets while the same is not possible with knowledge assets. To distinguish the financing choices for physical assets from those for knowledge assets, we show that the choice of access is irrelevant in the case of physical assets. Using the outside options in the case of physical assets (equation (11)) in (17), the first-order conditions for investment in the case of physical assets are given by

$$\begin{aligned} &\left[\beta + (\gamma + \beta_P)(1-\delta)\,\theta^S\right]\phi_e\left(e^{S*}\right) = 2 \\ &\left[\beta + \{(1+\gamma)(1-\delta)\,\beta_P + \delta\}\,\theta^F\right]\phi_e\left(e^{F*}\right) = 2 \end{aligned} \tag{20}$$

Proposition 4 (Full Access always Optimal with Physical Assets):
(i) Given α, δ, $e^{i*} < e^{iF}$, $i = S, F \quad \forall \delta, \beta_P, \beta, \theta^S, \theta^F$.
(ii) $\alpha^* = 1 \ \forall \delta, \beta_P, \beta, \theta^S, \theta^F$.

Part (i) of this proposition confirms the GHM result that in the case of physical assets, the essential distortion in incentives is that of under-investment compared to the first-best. Part (ii) of the Proposition, which is a key result, shows that access is not a choice variable in the case of physical assets since full access is always optimal.

Since the owner of a physical asset can hold-up the user of the asset by withdrawing access to the physical asset at date 2, this hold-up problem leads to both the scientist and the financier underinvesting compared to the first-best. Since increasing access enhances the incentives of both the scientist and the financier simultaneously by enhancing the productivity of their investments, an increase in access alleviates the under-investment by both agents. Since both agents underinvest for all parameter values, full access is always optimal.[16] In contrast, as we show in the next subsection, choosing access and ownership are both important for incentive purposes when financing choices involve knowledge assets.

As we argued in Section 3.6.1, CVC and IVC financing involve varying levels of knowledge sharing between the financier and the scientist. VC financing is extensive in information technology and biotechnology industries but considerably less in traditional, physical asset intensive industries (Gompers and Lerner, 1999). Thus, financing choices involving such differing levels of knowledge sharing are dominant in knowledge asset intensive industries but absent in physical asset intensive industries, which is consistent with the prediction that access is (not) a choice variable when financing choices involve (physical assets) knowledge assets.

4.2 Knowledge assets

In the case of knowledge assets, access provided ex-ante cannot be withdrawn ex-post. Therefore, with knowledge assets, (a) under-investment in tailoring complementary pieces of knowledge and (b) over-investment in stealing knowledge are both material concerns.

4.2.1 Determinants of under- and over-investment

Using the functional forms of the outside options for knowledge assets (8) in (15), we obtain the marginal value of each agent's investment on her share of the profits as

$$\Pi_i^i = -1 + \underbrace{0.5\beta\phi_e\left(e^i,\alpha\right)}_{\text{Marginal Value of the Joint Profit}} + \underbrace{0.5\left(1+\gamma\right)\left(1-\mu\right)\theta^i\phi_e\left(e^i,\alpha\right)}_{\text{Marginal Value of expected rents from a Duopoly}} + \underbrace{0.5\Delta^i\mu\theta^i\phi_e\left(e^i,\alpha\right)}_{\text{Marginal Value of expected rents from a Monopoly for } i} \tag{21}$$

where $\Delta^S = 1-\delta$ and $\Delta^F = \delta$.

[16] Note that this result would not be altered even if we were to adopt the bargaining model as in deMeza and Lockwood (1998). deMeza and Lockwood (1998) point out that the no-trade payoffs in GHM are really inside options. In contrast in DeMeza and Lockwood (1998), the no-trade payoffs are outside options that capture either agent's ability to sign some other contracts with parties outside the relationship. For their results, they analyze the case where both agent's outside options are really unproductive ($0.5R' > r'$). Extending their assumption to our setup, we find that both firms would always underinvest by a good margin irrespective of the outcome of the bargaining game, i.e., irrespective of whether the two agents split the joint output by half or one agent's outside option binds. Therefore, even in the DeMeza and Lockwood (1998) modification of GHM, access has no role to play since access should be optimally one.

Substituting (21) into (17), we get the following simplified first-order conditions for the second-best level of investments $\left(e^{S*}, e^{F*}\right)$ for knowledge assets

$$0.5\left[\beta+\left\{(1+\gamma)(1-\mu)+\Delta^{i}\mu\right\}\theta^{i}\right]\phi_{e}\left(e^{i*},\alpha\right)=1 \tag{22}$$

Proposition 5 (Determinants of under- and over-investment): Given α, δ, for $i = S, F$,

$$\left[(1+\gamma)(1-\mu)+\Delta^{i}\mu\right]\theta^{i} \lesseqgtr \beta \Leftrightarrow e^{i*} \lesseqgtr e^{iF}\forall\alpha \tag{23}$$

Corollary 1 (Marginal Effects on investment): Given α, δ, for $i = S, F$, $(i)\frac{de^{i*}}{d\mu} < 0$, $\frac{de^{iF}}{d\mu} = 0$; $(ii)\frac{de^{i*}}{d\gamma} > 0$, $\frac{de^{iF}}{d\gamma} = 0$; $(iii)\frac{d^2e^{i*}}{d\gamma d\mu} < 0$; $(iv)\frac{de^{i*}}{d\theta^i} > 0$, $\frac{de^{iF}}{d\theta^i} = 0$; $(v)\frac{de^{iF}}{d\beta} > \frac{de^{i*}}{d\beta} > 0$.

Proposition 5 and the adjoining corollary describe how investment incentives of the scientist and the financier — in particular their incentives to under- or over-invest compared to the first-best benchmark — depend upon (i) the strength of IP protection (μ); (ii) the intensity of product market competition (γ); (iii) how easily the scientist and the financier can expropriate each other's knowledge $\left(\theta^S, \theta^F\right)$; and (iv) the complementarity in the knowledge assets (β). The proposition follows using Proposition 3 together with (21) and $R_i = \beta\phi_i\left(e^i\right), i = S, F$.

The intuition for part (i) is as follows. Since the scientist gets to operate as a monopoly when she owns the idea and wins a law suit against the financier, an increase in IP protection enhances the likelihood of a monopoly but decreases that of a duopoly. Therefore, an increase in IP protection increases the marginal value of the expected rents that the scientist generates from a possible monopoly (see term 3 in (21)) but decreases that from a possible duopoly (see term 2 in (21)). However, since the scientist's investment in expropriation has the dual effect of increasing her duopoly profits as well as dampening the financier's duopoly profits, the effect of the second term dominates that of the third term, i.e. the effect of the rents from a possible duopoly dominates that from a possible monopoly. Since an increase in IP protection has no direct effect on the joint profit R, it either mitigates over-investment or accentuates under-investment compared to the first-best. A similar intuition applies for the financier's investment as well.[17]

An increase in the intensity of product market competition, ceteris paribus, enhances the marginal rents from the scientist's (financier's) investment by dampening the financier's (scientist's) profits (see term 2 in (21)). Since an increase in the intensity of product market competition has no direct effect on the joint profit, it either accentuates the scientist's and financier's over-investment or mitigates their under-investment compared to the first-best.

An increase in the intensity of product market competition increases the marginal effect of intellectual property protection. This interaction effect can be seen by examining the second term in (21). As argued before, an increase in IP protection decreases the likelihood of a duopoly. The marginal effect of this reduction in the likelihood of a duopoly is greater when the intensity of product market competition is greater. Therefore, an increase in IP protection decreases dispro-

[17] This is because unlike average values, which exhibit a zero-sum property, an identical effect of IP protection for both the scientist and the financer is possible with marginal values.

portionately the marginal value of the expected rents from a possible duopoly when the intensity of product market competition is greater. Therefore, an increase in the strength of IP protection *mitigates disproportionately* over-investment or *accentuates disproportionately* under-investment by the scientist and the financier when the intensity of product market competition is greater.

If the scientist can expropriate the financier's knowledge more easily, the marginal value of her share of the joint profit is higher since her expected rents from the duopoly and monopoly scenarios are greater (see terms 2 and 3 in (21)). Therefore, when the scientist can expropriate the financier's knowledge easily, the scientist's under-investment decreases or her over-investment increases. A similar intuition applies for the financier's investment as well.

An increase in the complementarity of knowledge assets enhances the marginal value of the joint profit more than the marginal value of either agent's share of the joint profit (compare term 1 in (21) with $TS_i = \beta\phi_e\left(e^i, \alpha\right)$). Therefore, an increase in the complementarity mitigates over-investment or accentuates under-investment compared to the first-best for both the scientist and the financier.

Proposition 6 (Incentive Effects Of Access And Ownership):
(a) $\frac{de^{i*}}{d\alpha} > 0\ \forall \delta, \beta, \mu, \gamma, \theta^S, \theta^F, i = S, F$

(b) $e^{S*}\left(\delta = 1\right) > e^{S*}\left(\delta = 0\right); e^{F*}\left(\delta = 1\right) < e^{F*}\left(\delta = 0\right)\ \forall \alpha, \beta, \mu, \gamma, \theta^S, \theta^F$

This result shows the contrasting effect of access and ownership on investment: ceteris paribus, changing access has a symmetric effect on the scientist's and financier's incentives to tailor their assets to each other as well as to expropriate the other's asset (part (a)). In contrast, ceteris paribus, changing ownership has an asymmetric effect since the incentives of the agent getting ownership are enhanced while those of the agent losing ownership get dampened (part (b)). Part (a) follows from the marginal product of investment being higher when access is higher (refer to (13) while part (b) follows from the fact that ownership enhances the marginal product of investment on the outside option as seen in part (b) of Proposition 1. Thus, the incentive effects of access and ownership contrast with each other: *access has a symmetric effect* on S's and F's incentives while *ownership has an asymmetric effect*.

Trade-offs underlying the financing choices: The above Proposition highlights the trade-offs between the various financing choices. To examine the trade-offs implied by the symmetric effect of access, we compare among CVC, IVC and angel financing. Since in each of these cases, the scientist owns the idea, these comparisons involve a changes in access without a change in ownership. Recall that CVC involves high access while IVC involves moderate access. Therefore, compared to IVC, CVC has the benefit of providing the scientist and the financier stronger incentives to tailor their knowledge to each other. However, compared to the IVC, CVC has the attendant cost that the scientist and the financier would invest more to expropriate each other's knowledge. In other words, compared to IVC, CVC has the benefit of providing the scientist and the financier stronger incentives to invest in developing the idea but the attendant cost of providing them greater incentives to waste costly resources in enhancing their respective bargaining positions by expropriating each other's knowledge. Since angel financing involves low access while IVC involves moderate access, these costs and benefits are similar when we compare angel financing to IVC. However, since CVC involves high access while angel financing involves low access, the costs and benefits mentioned above are more pronounced when we compare CVC to angel financing.

To examine the trade-off implied by the asymmetric effect of ownership, compare CVC to internal capital markets since this involves a change in ownership of the idea from the scientist to the corporation (which is the financier). Compared to CVC, in internal capital markets, the scientist has weaker incentives (i) to tailor her knowledge to that of the corporation's and (ii) to expropriate the same. The corporation, however, has stronger incentives to do both.

Corollary 2 (Adverse Effect of Ownership in Knowledge Assets):

$(1+\gamma)(1-\mu)\theta^F > \beta \Leftrightarrow e^{F*}(\delta = 1) > e^{F*}(\delta = 0) > e^{FF}$

Given the possibility of over-investment in knowledge assets, this result highlights that transferring ownership of the idea to the financier can have the *adverse effect* of encouraging over-investment by the financier. To understand the intuition behind this result, recall that the financier (scientist) owns the idea in the case of internal capital markets (CVC). Thus, in financing through internal capital markets, the corporation as the financier has the right to sue at date 2 if the scientist leaves the corporation after acquiring working as an employee from date 0 to date 2 and acquiring expertise over the corporation's asset. As a result, the corporation's threat to dispense with the scientist is more credible in internal capital markets than in the case of CVC. However, the corporation can't follow up on its threat of dispensing with the scientist *unless* it invests at date 1 to understand the scientist's idea *in its totality.* Thus, compared to CVC, in financing through internal capital markets, the corporation may divert more costly resources away from developing the idea in duplicating what the scientist already knows. This duplication represents the adverse effect of transferring ownership of the idea to the corporation.

Given this adverse effect of ownership, internal capital markets may not be optimal despite the idea being complementary to the corporation's knowledge assets, which is contrary to an important result in the Property Rights Theory that one agent controlling both assets is optimal when the assets are complementary to each other (see Proposition 2 (D) in Hart, 1995).

Note that this adverse effect of ownership does not materialize in the case of physical assets since both the scientist and the financier always underinvest compared to the first-best benchmark (see part (i) of Proposition 4). The intuition for this difference is as follows. Since access can be withdrawn ex-post to physical assets, the corporation's threat to dispense with the scientist is quite credible even if the corporation does not understand how to use the scientist's physical asset in its totality. As a consequence, the transfer of ownership does not induce the corporation to invest more in expropriation.

Corollary 3 (Irrelevance of ownership for incentives when IP protection is non-existent):

$\mu = 0 \Rightarrow \left[e^{i*}(\delta = 0) = e^{i*}(\delta = 1)\right], i \in S, F$

If IP protection is non-existent, ownership of knowledge assets has no effect on the incentives of the scientist or the financier. Since the scientist's and financier's outside options are unchanged due to a change in ownership if IP protection is non-existent (see Proposition 2), it follows that ownership has no effect on incentives in this case.

Corollary 4 (Effect of ownership when expropriabilities are equal):
$\theta^S = \theta^F \Rightarrow \left[e^{S*}(\delta = 0) - e^{S*}(\delta = 1)\right] = \left[e^{F*}(\delta = 1) - e^{F*}(\delta = 0)\right]$

If the scientist's and financier's intrinsic abilities are equal and the technological characteristics of the scientist's knowledge are identical to those of the financier, then the decrease in the scientist's investment when the idea is transferred to the financier equals the increase in the financier's investment. Thus, in this case, a change in ownership has an effect of equal magnitude, albeit asymmetric, on the scientist's and financier's investments. The intuition for this result can be obtained by examining the expression for Π_i^i — the marginal effect of investment on the scientist's and financier's share of joint surplus in (21). The third term in (21), which is the only term affected by ownership, is identical in magnitude for Π_S^S and Π_F^F if $\theta^S = \theta^F$. Hence, a change in ownership has the same magnitude of effect on the scientist's and financier's investments.

4.3 Optimal Financing Choice

The optimal financing choice, which corresponds to the optimal combination of access and ownership, maximizes the joint surplus. Since access has a symmetric effect on incentives, regulating access brings both F's and S's incentives closer to first-best *when both agents over- or under-invest* compared to the first-best. In contrast, changing ownership helps bring both the financier's and the scientist's incentives closer to first-best when one agent over-invests while the other under-invests. Thus, optimally choosing access and ownership *simultaneously* alleviates the problems of (a) under-investment in tailoring the idea and the financier's assets to each other, and (b) over-investment in expropriating them.

Proposition 7 (Product Market Competition, IP Protection and Optimal Access):
$(i)\frac{d\alpha^*}{d\mu} \geq 0;\ (ii)\frac{d\alpha^*}{d\gamma} \leq 0;\ (iii)\frac{d^2\alpha^*}{d\mu d\gamma} > 0\ \ \forall \delta, \gamma, \mu, \theta^S, \theta^F, \beta$

Proposition 7 shows that, ceteris paribus, the optimal level of access increases weakly with an increase in the strength of IP protection and a decrease in the intensity of product market competition. Furthermore, an increase in the strength of IP protection increases the optimal level of access *disproportionately* when the intensity of product market competition is higher.

Recall from Proposition 5 that, ceteris paribus, compared to an environment with weak IP protection, the scientist and the financier either overinvest less or underinvest more in an environment with strong IP protection. Consider first the case where both the financier and the scientist underinvest compared to the first-best benchmark. Since access has a symmetric effect on incentives, *increasing* access enhances *both agent's incentives* and brings their investments closer to the first-best. Since the extent of under-investment is greater when IP protection is stronger, access should optimally be higher in such an environment. Now consider the case where both the financier and the scientist overinvest compared to the first-best. In this case, *decreasing* access lowers *both agent's incentives* and brings their investments closer to the first-best. Since the extent of over-investment is greater when IP protection is weaker, access should optimally be lower in such an environment. Therefore, the optimal level of access increases with an increase in the strength of IP protection.

Similarly, recall from Proposition 5 that, ceteris paribus, compared to an industry where product competition is intense, the scientist and the financier either overinvest less or underinvest more in an industry where the product market competition is moderate. Following arguments identical to the above, it follows that the optimal level of access should be higher when the product market competition is moderate than when it is intense.

Since an increase in the intensity of product market competition increases the marginal effect of intellectual property protection on the scientist's and financier's investments, therefore the optimal level of access should increase disproportionately with the strength of IP protection in industries where the intensity of product market competition is intense. The arguments leading up to this result are very similar to those provided above.

Proposition 8 (Complementarity in Knowledge Assets and Optimal Access):

$\frac{d\alpha^*}{d\beta} \geq 0 \;\; \forall \delta, \gamma, \mu, \theta^S, \theta^F, \beta$

Ceteris paribus, the optimal level of access increases weakly with an increase in the complementarity of the knowledge assets. The intuition for this result is also similar to that described in Proposition 6 above — since an increase in the complementarity of knowledge assets either mitigates the over-investment or accentuates the under-investment by both the financier and the scientist, an increase in the complementarity increases the optimal level of access.

Proposition 9 (Ability to Expropriate and Optimal Access):

$\frac{d\alpha^*}{d\theta^i} \leq 0 \;\; \forall \delta, \gamma, \mu, \theta^S, \theta^F, \beta$

Ceteris paribus, the optimal level of access increases weakly with a decrease in the scientist's and corporation's abilities to expropriate. The intuition for this result is similar to that described in Proposition 7 above — since a decrease in the scientist's (financier's) ability to expropriate the financier's complementary asset (scientist's idea) either mitigates the extent of her over-investment or accentuates the extent of her under-investment, an decrease in such ability increases the optimal level of access.

Proposition 10 (Irrelevance of ownership when Expropriabilities are equal):

$\theta^S = \theta^F \Rightarrow TS^* (\delta = 1) = TS^* (\delta = 0) \;\; \forall \alpha, \gamma, \mu, \beta$

The above Proposition states that if the scientist's and financier's intrinsic abilities are equal and the technological characteristics of the scientist's knowledge are identical to those of the financier, then the allocation of ownership of the idea is irrelevant. In this case, the joint surplus is the same irrespective of whether the scientist or the financier owns the idea. Consequently, the optimal level of access is the same too irrespective of who owns the idea. The intuition for this result follows using Corollary 3, which shows that the change in investments of the scientist and the financier due to a change in ownership of the idea are equal in magnitude, albeit asymmetric. Therefore, the change in the scientist's contribution to the equilibrium level of the surplus TS^* equals in magnitude the change in the financier's contribution to the equilibrium level of the surplus. However, since the signs of these effects are opposite to each other, it follows that the change in surplus due to a change in ownership is identically zero.

Proposition 11 (Irrelevance of ownership without IP Protection):
$\mu = 0 \Rightarrow TS^*(\delta = 1) = TS^*(\delta = 0) \;\; \forall \alpha, \gamma, \mu, \beta$

If IP protection is non-existent, then ownership of the idea is irrelevant since it has no effect on the incentives of the scientist or the financier. This result highlights that for ownership of knowledge assets to be economically relevant, IP protection is paramount.

Proposition 12 (Expropriability and Optimal Access): Let $\underline{\lambda} = \frac{\phi_\alpha^{\min}|\phi_{ee}|}{\phi_{e\alpha}}$ and $\overline{\lambda} = \frac{\phi_\alpha^{\max}|\phi_{ee}|}{\phi_{e\alpha}}$. Clearly $\underline{\lambda} < \overline{\lambda}$. Then

(i) $\frac{1}{\beta+\{\delta\mu+(1+\gamma)(1-\mu)\}\theta^F} + \frac{1}{\beta+\{(1-\delta)\mu+(1+\gamma)(1-\mu)\}\theta^S} > \frac{1}{\beta} - \underline{\lambda} \Leftrightarrow \alpha^* = 1$

(ii) $\frac{1}{\beta+\{\delta\mu+(1+\gamma)(1-\mu)\}\theta^F} + \frac{1}{\beta+\{(1-\delta)\mu+(1+\gamma)(1-\mu)\}\theta^S} < \frac{1}{\beta} - \overline{\lambda} < 0 \Leftrightarrow \alpha^* = 0$

(iii) $\frac{1}{\beta} - \overline{\lambda} < \frac{1}{\beta+\{\delta\mu+(1+\gamma)(1-\mu)\}\theta^F} + \frac{1}{\beta+\{(1-\delta)\mu+(1+\gamma)(1-\mu)\}\theta^S} < \frac{1}{\beta} - \underline{\lambda} \Leftrightarrow 0 < \alpha^* < 1$

This result shows how the optimal level of access varies with the ease with the scientist and the financier can expropriate each other's knowledge. When both the scientist and the financier cannot (can) expropriate each other's knowledge easily, full (zero) access is optimal while for intermediate values, interior levels of access is optimal. This follows directly using the symmetric effect of access together with the fact that when knowledge becomes easier to expropriate, the incentive to over-invest is greater.

Proposition 13 (Expropriability and Optimal Ownership):
(i) If $\theta^S + \theta^F \leq \frac{0.25\beta}{2+2\gamma-2\gamma\mu-\mu}$ then $\theta^S \gtreqless \theta^F \Rightarrow TS^*(\delta = 1) \lesseqgtr TS^*(\delta = 0)$.
(ii) If $\theta^S + \theta^F > \frac{0.25\beta}{2+2\gamma-2\gamma\mu-\mu}$ then $\theta^S \gtreqless \theta^F \Rightarrow TS^*(\delta = 1) \gtreqless TS^*(\delta = 0)$.

When the scientist's and the financier's knowledge assets are tacit and difficult to expropriate (part (i)), the scientist should optimally own the idea when she is more capable/ productive than the financier.[18] When their knowledge assets are codified and easy to expropriate, the result is reversed.

[18] Note that we are interpreting $\theta^S + \theta^F \leq \frac{0.25\beta}{2+2\gamma-2\gamma\mu-\mu}$ as the knowledge assets being relatively more difficult to expropriate and $\theta^S \gtreqless \theta^F$ as differences in the relative abilities of the scientist and the financier. It is easy to see that we could separately parametrize the characteristics of the knowledge assets and the intrinsic abilities/ productivities of the scientist and the finanicer and obtain results with an identical interpretation.

The intuition for this result is as follows. When the scientist is more capable/ productive than the financier, a change in ownership affects the scientist's incentives more than that of the financier. Therefore, when both agents underinvest compared to the first-best (i.e. when their knowledge assets are relatively tacit and difficult to expropriate as in part (i)), giving the scientist ownership is optimal since this alleviates the problem of under-investment better by enhancing the scientist's incentives more than dampening the incentives of the financier. In contrast, when both agents overinvest compared to the first-best (i.e. when their knowledge assets are relatively codified and easy to expropriate as in part (ii)), giving the financier ownership is optimal when the scientist's ability to expropriate is greater than that of the financier. This is because giving the financier ownership alleviates the problem of over-investment better by dampening the scientist's incentives more than enhancing the incentives of the innovator.

Subramanian (2008) finds evidence consistent with the predictions in Proposition 6, 9 and 13 above. Using a sample of alliances and majority joint ventures (JVs) together with patent citation based variables to proxy expropriability of knowledge, the study finds that *access* through a technology transfer, joint R&D or license agreement is more likely in an alliance when the expropriability of both partner firms is greater, which is consistent with the symmetric effect of access on incentives that underlies the predictions in Proposition 6(i) and 9. The study also finds that a *difference in ownership* in the form of a majority JV rather than an alliance is likely when the ability of one partner firm increases while the other decreases. This evidence is consistent with the asymmetric effect of ownership on incentives which underlies the predictions in Propositions 6(ii) and 13.

5 Empirical Implications

We now detail the empirical implications that follow from the results. Existing empirical evidence is consistent with some of the predictions. However, most of the empirical predictions that we discuss below are novel and are yet to be empirically tested. To test the new predictions, data on IVC, CVC and angel financing in the US can be combined with proxies for IP protection μ, product market competition γ, complementarity in the assets β and measures for expropriability of knowledge. Cohen, Nelson and Walsh (2000) generate survey measures for the level of IP protection and the importance of complementarity assets in over 140 industries at the 4-digit SIC level, which can be used to proxy μ and β respectively. Herfindahl index based measures can be used to construct proxies for the degree of product market competition γ. Proxies for the ability to expropriate knowledge assets, θ^S and θ^F, can be constructed using self-citations as in Trajtenberg, Jaffe and Henderson (1992) and Subramanian (2008).[19]

Proposition 7 provides the following prediction:

[19] Since citations to patents track the trail of spillovers from existing knowledge, they are useful to construct proxies for expropriability of knowledge. In particular, citations made to a firm's own patents measure the extent to which the originating innovation represents appropriation of benefits to its predecessors housed in the same firm.

Implication 1: As IP protection becomes stronger, CVC financing becomes more likely than IVC financing.

This prediction finds support in evidence documented by Gans, Hsu and Stern (2002). They estimate using a novel dataset of 118 start-up companies that firms with greater intellectual property protection (start up firms having at least one patent associated with their technology) are 23% more likely to pursue a cooperative strategy with incumbent corporations than firms that have no patents associated with their technology. Furthermore, Lerner and Merges (1998) document the widespread use of research alliances between pharmaceutical firms and biotech startups while Bhide (2000) finds the salience of IVC financing in Internet, Software and Semiconductors. This is consistent with the above prediction since Cohen, Nelson and Walsh (2000) find that patent protection is stronger in Drugs and Biotechnology than in Internet, Software and Semiconductors.

Proposition 7 also generates the following new predictions:

Implication 2: As IP protection becomes stronger, IVC and CVC financing become more likely than angel financing.

Implication 3: As product market competition becomes more intense, IVC financing becomes less likely than angel financing but more likely than CVC financing.

Implication 4: Compared to industries where product market competition is moderate, an increase in IP protection leads to IVC financing becoming *disproportionately* more likely than angel financing and *disproportionately* less likely than CVC financing in industries where product market competition is intense.

Proposition 8 implies that:

Implication 5: When the complementarity between the financier's and scientist's knowledge assets is greater, CVC financing would be more likely than IVC financing.

Consistent with this prediction, Gans, Hsu and Stern (2002) find in their study of start-up firms that firms with a greater need for the complementary assets of a corporation were more likely to pursue a cooperative strategy with incumbent corporations than those firms that without such need for complementary assets.

Proposition 8 predicts that:

Implication 6: When the complementarity between the financier's and scientist's knowledge assets is greater, IVC financing would be more likely than angel financing.

Proposition 9 implies that:

Implication 7: An increase in the scientist's or the financier's ability to expropriate their partner's knowledge leads to IVC financing becoming less likely than angel financing and more likely than CVC financing.

Contrasting Propositions 7-9 with Proposition 10 above reveals an important implication of the contrasting incentive effects of access and ownership. As we noted in Propositions 7-9, the strength of IP protection μ, the intensity of product market competition γ and the level of complementarity in the knowledge assets β affect the optimal level of access for all values of θ^S and θ^F, even when $\theta^S = \theta^F$. In contrast, as Proposition 10 shows, if the scientist's and financier's ability to replicate are identical and their knowledge assets are identical in nature, then none of these macro-level variables have any effect on the optimal allocation of ownership.

Implication 8: Without differences in micro firm-level variables such as (a) the relative abilities of the innovator and the financier or (b) the nature of their knowledge assets, financing ideas inside or outside the boundaries of an existing firm is irrelevant irrespective of differences in industry level competition or the level of IP protection. Even without differences in these micro firm-level variables, the choice of IVC, CVC and angel financing of startups is influenced by industry level variables such as the intensity of product market competition, the level of IP protection and the complementarity between the knowledge assets.

Figure 3 depicts pictorially the optimal financing choice as a function of how easily the scientist and the financier can expropriate each other's knowledge. The boundaries for the optimal level of knowledge sharing as implied by Proposition 12 are plotted in Figure 2. Figure 3 is obtained by super-imposing implications of Proposition 13 over Figure 2.

Since internal capital markets corresponds to the financier owning the idea, Proposition 13 predicts that Internal capital markets is optimal in regions A, C, E and H. In other words, Figure 3 implies that

Implication 9: (a) Internal capital markets is optimal in two different scenarios: (i) when the scientist and the financier's knowledge is simple and codified and the scientist has greater ability than the financier (region H in Figure 3); and (ii) when their knowledge is relatively complex and tacit and the scientist has lesser ability than the financier (regions A, C and E in Figure 3).

As Proposition 12 predicts, the optimal level of knowledge sharing in internal capital markets itself varies with the nature of knowledge. Thus,

Implication 9: (b) As knowledge of both agents becomes relatively more tacit and complex, the optimal of knowledge sharing in internal capital markets increases.

Since a start-up corresponds to the scientist owning the idea, Proposition 13 predicts that the creation of a startup is optimal in regions B, D, F and G in Figure 3. When the scientist and the financier's knowledge is quite simple and codified and the financier has greater ability than the scientist (region G in Figure 3), angel financing is optimal. Thus Figure 3 implies that

Implication 10: Angel financing is optimal in two scenarios: (i) when the scientist and the financier's knowledge is very simple and the financier has greater ability than the scientist (region G in Figure 3); and (ii) when their knowledge is simple but less than that in region G but the scientist has greater ability than the financier (region F in Figure 3).

Implication 11: IVC financing is optimal when their knowledge is reasonably tacit and complex and the scientist has greater ability than the financier (region D in Figure 3)

Implication 12: CVC financing is optimal when their knowledge is extremely tacit and complex and the scientist has greater ability than the financier.

Combining implications 11 and 12, it follows that

Implication 13: VC financing — whether CVC or IVC — is optimal only if the scientist has greater ability than the financier.

Laws that change Intellectual Property protection can also be employed as natural experiments to test the predicted effect of IP protection. For example, the Cooperative Research and Technology Enhancement (CREATE) Act, which was enacted as a law by US Congress in March 2004, allows a patent application to be approved even if it involves collaborators from more than one organization (Meagher and Copeland, 2006). In the past, prior work by one of the partners in a joint research program could be used as prior art to deny a patent for discoveries made under the joint research program. The CREATE act removes such obstacles for collaborative research. In the framework of this paper, the CREATE act enhances protection to intellectual property generated through collaborative research by allowing them to be patented. Similarly, the World Trade Organization (WTO)'s agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS) requires, starting January 2005, that developing countries must implement product patents on drugs. Thus, the CREATE act and TRIPS agreement provide time-series proxies where μ increased due to the enactment of these laws.

6 Robustness

Before concluding, let us examine how robust our results are to changes in the assumptions. This would enable us to highlight the assumptions that are crucial and those that are not.

6.1 Incomplete Contracts

Kaplan and Stromberg (2003) document that VCs write detailed contracts specifying the allocation of cash flows and control rights. Therefore, in this section, we show in an extension of the basic model that our results are robust even if we allow the scientist and the financier to contract on the division of cash flows and the allocation of control.

6.1.1 Control rights

Following Aghion and Bolton (1992), to model the allocation of control rights, we allow for an action $\omega \in [\underline{\omega}, \overline{\omega}]$ that can be taken either by the scientist or the financier. For example, the action could be recruitment of employees, in which case ω denotes the number of employees that are decided to be recruited.[20] To allow for the action to affect the investments that are made at

[20] What if the action over which control is to be contracted upon is the act of repudiation at date 2? In other words, if the scientist has control, then the financier cannot withdraw from the relationship and pursue his outside opportunities at date 2. Since such control rights essentially stem from ownership of the idea, contracting on the

date 1, we assume that the action is taken at date 0.5, i.e. in between date 0 and date 1. We assume that though the action is not verifiable, the right to implement this action is contractible at date 0. Say $\Omega = 0$ ($\Omega = 1$) denotes the scientist (financier) having the right to implement this action. Note that we could assume that if the scientist (financier) owns the idea, then she (he) has the right to implement the action. However, since we would like to show that our results are robust when we allow for contracting on control rights for any of the organizational modes for financing the idea, we allow for control rights independent of ownership. Given α, δ, Ω the equilibrium level of the joint surplus TS^*, the scientist's share Π^{S*} and the financier's share Π^{F*} are given by

$$\begin{aligned} TS^* &= R\left(e^{S*}, e^{F*}, \omega, \alpha\right) - \sum_{i \in S,F} e^{S*}\left(\omega\left(\Omega, \alpha\right), \alpha, \delta\right) \quad (24) \\ \Pi^{S*}\left(\omega\right) &= 0.5\left[R\left(e^{S*}, e^{F*}, \omega, \alpha\right) + r^S\left(e^{S*}, e^{F*}, \omega, \alpha\right) - r^F\left(e^{S*}, e^{F*}, \omega, \alpha\right)\right] - e^{S*}\left(\omega\left(\Omega, \alpha\right), \alpha, \delta\right) \\ \Pi^{F*}\left(\omega\right) &= 0.5\left[R\left(e^{S*}, e^{F*}, \omega, \alpha\right) + r^F\left(e^{S*}, e^{F*}, \omega, \alpha\right) - r^S\left(e^{S*}, e^{F*}, \omega, \alpha\right)\right] - e^{F*}\left(\omega\left(\Omega, \alpha\right), \alpha, \delta\right) \end{aligned}$$

We assume that a higher level of access enhances the efficiency of the action and that a higher level of the action enhances the efficiency of the investments, i.e.

$$\phi_{\alpha\omega} > 0, \phi_{e\omega} > 0 \quad (25)$$

Given this modified setup, at date 0, the scientist and the financier choose the optimal allocation of control rights Ω apart from the financing choice (α^*, δ^*) itself. Say, ω^{FB} denotes the first-best choice of action that maximizes the joint surplus. The second-best actions are chosen by the scientist and the innovator to maximize their respective shares of the joint surplus. Say that ω^S $\left(\omega^F\right)$ denotes the optimal action implemented by the scientist (innovator). Thus,

$$\begin{aligned} \omega^{FB} &\equiv \arg\max_{\omega} \; TS^*\left(\omega\right) \\ \omega^S &\equiv \arg\max_{\omega} \; \Pi^{S*}\left(\omega\right) \\ \omega^F &\equiv \arg\max_{\omega} \; \Pi^{F*}\left(\omega\right) \end{aligned}$$

To focus on the situations where a conflict of interest arises between the actions implemented by the scientist and the financier, we assume that the first-best level of action has an interior solution, i.e. $\omega^{FB} \in \left(\underline{\omega}, \overline{\omega}\right)$.

Lemma 2 (Choice of Action): $\Omega = 0 \Rightarrow \omega \in \left\{\omega^{FB}, \omega^S\right\}, \Omega = 1 \Rightarrow \omega = \omega^F$.

If the scientist has the control right, i.e. $\Omega = 0$, then $\omega \in \left\{\omega^{FB}, \omega^S\right\}$. If the financier can make a private transfer at date 0.5 without violating his IR constraint, then the scientist chooses ω^{FB}. If the financier cannot make a private transfer to the scientist without violating his IR constraint,

control over such action is economically equivalent to contracting on the ownership of the idea, which we have already considered.

then ω^{FB} is not implementable and so the scientist will choose ω^S. Since the scientist is financially constrained, the financier having the control right, i.e. $\Omega = 1$, implies that the financier will choose ω^F. Thus, as argued by Aghion and Bolton (1992), the scientist and the financier would differ in their choice of actions.

Proposition 14 (Conflict of Interests in implementing the Action):(i) If $\omega^S \leq \omega^{FB} \Longleftrightarrow \omega^F \geq \omega^{FB}\ \forall \alpha, \delta$ (ii) If $\omega^S > \omega^{FB} \Longleftrightarrow \omega^F < \omega^{FB}\ \forall \alpha, \delta$.

If the scientist's action is below (above) the first-best benchmark level of action ω^{FB}, then the financier's action would be above (below) this first-best benchmark.

Now, allowing for the allocation of control rights, we examine the key result using the basic model, i.e. access has a symmetric effect while ownership has an asymmetric effect on the scientist's and financier's incentives (Proposition 6).

Proposition 15 (Allowing for Control Rights, Incentive Effects Of Access And Ownership):
(a) (i)For $i \in \{S, F\}$, $\frac{de^{i*}(\omega = \omega^{FB})}{d\alpha} > 0\ \forall \alpha, \delta, \beta, \mu, \gamma, \theta^S, \theta^F$

(ii) There exists $\underline{\mu} < 0.5$ such that for $i, j \in \{S, F\}$ $\mu > \underline{\mu} \Rightarrow \frac{de^{i*}(\omega = \omega^j)}{d\alpha} > 0.\ \forall \alpha, \delta, \beta, \gamma, \theta^S, \theta^F$.

(iii) There exists $\overline{\theta} > 0.5$ such that for $i, j \in \{S, F\}$ $\theta^F < \overline{\theta} \Rightarrow \frac{de^{i*}(\omega = \omega^j)}{d\alpha} > 0\ \forall \alpha, \delta, \beta, \mu, \gamma, \theta^S$ and $\theta^S < \overline{\theta} \Rightarrow \frac{de^{i*}(\omega = \omega^j)}{d\alpha} > 0\ \forall \delta, \alpha, \beta, \mu, \gamma, \theta^F$.
(b) $e^{S*}(\delta = 1) > e^{S*}(\delta = 0)\,; e^{F*}(\delta = 1) < e^{F*}(\delta = 0)\ \forall \alpha, \Omega, \beta, \mu, \gamma, \theta^S, \theta^F$

If the first-best action is implemented, then increasing access always increases the incentives of the scientist and the financier (part (a)(i)). In contrast, if the first-best action is not implemented, then for most parameter values, the symmetric effect of access holds (parts (a)(ii) and (iii)). Part (a)(ii) shows that this is true for all other parameter values as long as IP protection is above a low threshold. Part (a)(iii) shows this is true for all other parameter values as long as the ease of stealing knowledge is lower than a high threshold.

If IP protection is non-existent (i.e.$\mu = 0$), product market competition is extreme (i.e. $\gamma = 1$), complementarity between the assets is non-existent (i.e. $\beta = 1$), *and* the ease of expropriating knowledge are extremely skewed (i.e. $\theta^S = 0$ and $\theta^F = 1$ or $\theta^S = 1$ and $\theta^F = 0$), then increasing access can lead to a decrease in the scientist's and financier's incentives if the first-best action cannot be implemented. This is because the second-best actions maximize only the respective shares of the joint surplus. If the scientist cannot expropriate the financier's knowledge but the financier can very easily do so, then an increase in access leads the scientist to implement a lower level of the action to ensure that the scientist does not end up losing rents to the financier. Since we have assumed that a lower level of the action makes the investments less productive, an increase in access lowers incentives for such extreme values. A similar intuition applies for the financier's action too.

With the restrictions on the parameter values, all the other comparative statics above remain unaltered. These results are omitted here for brevity and are available from the author on request.

6.1.2 Cashflow and Control rights

In order to allow for the allocation of cashflow rights, we have to allow for a portion of the joint profits to be verifiable. Therefore, say that of the joint profit S, a proportion $s = \xi S$ is verifiable and the remaining $R = (1-\xi) S$ is non-verifiable. Now, let us say that the financier offers the scientist an optimal compensation contract $q^*(s)$ at date 0, where s is the contractible portion of the joint profits. Given this modification, the scientist's and the financier's share of the non-verifiable portion of the joint surplus are Π^{S*} and Π^{F*} respectively.

The optimal financing choice together with the optimal allocation of control rights $(\alpha^*, \delta^*, \Omega^*)$ and the scientist's compensation contract $q^*(s)$ therefore solve the following optimization problem:

$$((\alpha^*, \delta^*, \Omega^*), q^*(s)) \equiv \underset{(\alpha,\delta,\Omega),w(s)}{\arg\max} \left[\Pi^{F*} + s - q^*(s)\right] \tag{26}$$

subject to the scientist's participation constraint,

$$\Pi^{S*} + q^*(s) \geq U, \tag{27}$$

and the incentive compatibility constraint,

$$(\alpha^*, \delta^*, \Omega^*) = \arg \max_{(\alpha,\delta,\Omega)} \left[\Pi^{S*} + \Pi^{F*} + s\right] \tag{28}$$

In constraint (27), the variable U denotes the scientist's reservation payoff. Constraint (28) is the constraint that the optimal financing choice together with the optimal allocation of control rights maximizes the net surplus (which now comprises both the verifiable and non-verifiable cashflows).

Proposition 16 (Financing Choice allowing for cashflow and control rights): If the financier is not liquidity constrained and the scientist has the bargaining power at date 0, the maximization problems (26) and (28) are equivalent.

Thus, if we allow for a portion of cash flows to be verifiable and provide for its contractual allocation between the scientist and the financier apart from allocating control rights between them, the maximization problem with and without the cashflow contract are equivalent. We have already shown that our results are robust for almost the entire range of parameter values when we allow for contracting on control rights. Therefore, we can conclude that our results remain robust to the allocation of both cashflow and control rights between the scientist and the financier.

6.1.3 Joint Ownership with the Option to Sell

Maskin and Tirole (1999), in their critique of incomplete contracts, argue that the inefficiency of under-investment stemming from incompleteness in contracts can be rectified through a "mechanism", which involves the following contract. The contract is registered with a court. Both agents own the asset, and so neither party can use the asset without the consent of the other. At date 2, however, one party, drawn at random with equal probabilities, receives the right to sell his/ her share in the joint venture at a pre-specified striking price. If a party exercises his exit option, the other party pays a tax t to the community of citizens. They show that in the absence of collusion and renegotiation, is the tax t is large enough, neither agent underinvests and the first-best level of investment results.

We now show in our setting that such a mechanism cannot solve the twin-problems of under- and over-investment. As in Hart (1995) and Maskin and Tirole (1999), we consider joint ownership to correspond to the case where the scientist (financier) cannot use the idea without the consent of the financier. Therefore, under joint ownership, each agent's outside option equals the value when he/ she does not own the idea. Denoting joint ownership by $\delta = J$, we have

$$\begin{aligned} r^S(\delta = J) &= r^S(\delta = 1) \\ r^F(\delta = J) &= r^F(\delta = 0) \end{aligned} \tag{29}$$

According to Maskin and Tirole (1999), the exercise price for $S(F)$ should be set equal to S's (F's) share of the total output when $S(F)$ does not invest while $F(S)$ makes the first-best level of investment.[21] Therefore, the Maskin and Tirole (1999) exercise prices in our setting would be equal to

$$\begin{aligned} p^S &= 0.5\left[R\left(0, e^{FF}\right) + r^S\left(0, e^{FF}, J\right) - r^F\left(0, e^{FF}, J\right)\right] \\ p^F &= 0.5\left[R\left(e^{SF}, 0\right) + r^F\left(e^{SF}, 0, J\right) - r^F\left(e^{SF}, 0, J\right)\right] \end{aligned} \tag{30}$$

We show in the Proposition below that this mechanism *does not* lead to first-best levels of investment by both the scientist and the financier.

Proposition 17 (Investment Choices with the Maskin and Tirole (1999) mechanism): Consider the Maskin and Tirole (1999) mechanism described above. The investment under this mechanism, which we denote by $\left(e^{S*}(M), e^{F*}(M)\right)$ is not necessarily efficient.

$$\left(e^{S*}(M), e^{F*}(M)\right) \neq \left(e^{SF}, e^{FF}\right) \tag{31}$$

The Maskin and Tirole (1999) mechanism is designed to weed out underinvestment due to incompleteness of contracts in the traditional GHM framework. Recall that in the GHM framework agents choose investments within the relationship, which has spillover effects on their outside options. Therefore, an agent's outside option corresponds to zero investment within the relationship by that agent. As a result, if either agent under-invests compared to the first-best level, the other agent finds it individually rational to exercise the option to sell. Consequently, both agents choose the efficient level of investment.

[21] If the output generated by the buyer and the seller is $v(b, s)$ and the first-best investment levels are given by (b^*, s^*), then according to Maskin and Tirole (1999), the exercise prices for the buyer and the seller should be set equal to $p^B = 0.5v(0, s^*)$ and $p^S = 0.5v(b^*, 0)$, where the factor 0.5 results from 50 : 50 Nash Bargaining since the outside options for both the buyer and the seller are zero under joint ownership.

In contrast, in the setup modeled here, agents' investment *directly affect* not only the joint profit but *also their outside options*. As a result, overinvestment is possible too. Given the possibility of overinvestment, it is not always optimal for the scientist (financier) to exercise her outside option at the exercise prices given by (30) in the scenario that the financier's (scientist) investment deviates from the first-best level.

6.2 Reciprocal Access

While the financing choices studied here naturally lead us to model access as reciprocal, this assumption warrants discussion.

If access can be contracted upon, then the scientist and the financier receiving differing levels of access improves efficiency compared to access being reciprocal.[22] In Appendix A.3, I analyze this scenario and show that an increase in the access that the scientist (financier) receives increases her (his) investment but has no effect on the financier's (scientist's) investment. Thus, even in the scenario where access is not reciprocal, the effect of access on incentives is not zero-sum as it is with ownership of the idea. In fact, in this case, as long as the access recieved by both the scientist and the financier moves in the same direction, the symmetric effect of access is still obtained. Thus, the symmetric effect of access on incentives is *not* a direct consequence of the reciprocity in access; instead, the symmetric effect results due to the non-zero sum nature of access in providing incentives.

As a practical matter though, access is very difficult to describe qualitatively and/ or measure quantitatively (Rajan and Zingales, 1998). As a consequence, access is generally difficult to verify and enforce. Given the lack of contractibility of access, it can be shown in a repeated game framework that the promised level of access becomes self-enforcing when such access is reciprocal. More specifically, given reciprocal access, if in any period S deviates by offering access lower than the promised level, then F can observe the same and will retaliate by doing so. Since the assets are complementary to each other, such deviations become quite costly to S through a reduction in the joint profits. In fact, it can be shown that for a broad range of the parameter values $\mu, \gamma, \theta^S, \theta^F$ and β, the reneging temptations for either the scientist or the financier are strictly lower when the promised access is reciprocal than when the scientist promises a higher level access to the financier or vice-versa. The static model presented here can be regarded as a reduced form abstraction of the repeated interactions taking place between firms in reality, where the assumption of reciprocal access would be considerably less rigid than it possibly is a static setting.

[22] Compared to the scenario where the access received by the scientist and the financier differ from each other, maximizing under reciprocal access corresponds to a constrained optimization where the access received by the scientist is kept equal to the access received by the financier. Since the value of the objective function without constraints is at least as much as that obtained with the constraint, clearly two different levels of access would dominate.

7 Conclusion

This paper developed an incomplete contracting theory of financing choices by integrating the incentive effects of *knowledge sharing* in corporate VC, independent VC, and angel financing together with the incentive effects of financing ideas inside or outside the boundaries of an existing firm. By distinguishing the financing of knowledge assets from that for physical assets, the theory rationalizes differences in the pattern of financing choices in these industries. The theory also provides several predictions about how the optimal financing choice varies with the strength of IP protection, the intensity of product market competition, the complementarity between the financier's and innovator's knowledge as well as the nature of their knowledge assets.

Though we examined the robustness of our results by allowing for allocation of control rights in Section 6.1, this was not the focus of the study. The framework employed here can be extended to study the optimal allocation of control rights in CVC vis-à-vis that in IVC or angel financing. More generally, the interaction between the nature of control rights and the financing choice is a potential area for future research.

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Appendix A – Additional Robustness Checks

A.1 Changing the Bargaining Model

In this Appendix, we show that the problem of over-investment and under-investment by both the Scientist and the Corporation would exist even if the Bargaining model for deciding the split of surplus at date 2 is changed. Then, we proceed to argue that the analysis in the paper would remain unaltered in this case.

As an alternative model of bargaining, let us use the alternating-offers protocol of Rubinstein (1982) that is employed by De Meza and Lockwood (1998) to question the generality of the Grossman-Hart-Moore results on ownership. As specified in Section 3.2, the bargaining for the surplus R occurs at date 2 after the contract has been signed at date 0 and after the investments are already sunk by both the scientist and the financier.

Bargaining occurs over multiple rounds $k = 1, 2, \ldots$. At the beginning of the first round, either the Scientist(S) or the Financier (F) is selected to be the proposer with probability 0.5. If the proposer is agent i, he proposes a split x_i so that S gets x_i, while F gets $R - x_i$. After agent i proposes, the responder $j \neq i$ has three choices. First, j can accept the proposal in which case the bargaining game ends. Second, j can reject the proposal in which case both agents get zero over that round and bargaining proceeds to the next round where j gets to make a proposal. Third, j could choose to terminate the bargaining process, in which case both S and F are obliged to pursue their own opportunities individually. In this case, S and F get their outside options r^S and r^F respectively. We allow only the responders to terminate the bargaining process since this ensures uniqueness of the solution to this bargaining game. Finally, the discount factor for both agents is $\tau < 1$.

The realized payoffs to S and F in equilibrium depend upon whether their outside options bind or they are slack. The realized payoffs to S and F, v^S and v^F, respectively are as follows:

$$\left(v^S, v^F\right) = \begin{cases} (0.5R, 0.5R) & \text{if} \quad r^S \leq 0.5R \text{ and } r^F \leq 0.5R \\ \left(R - r^F, r^F\right) & \text{if} \quad r^S > 0.5R \text{ and } r^F \leq 0.5R \\ \left(r^S, R - r^S\right) & \text{if} \quad r^S \leq 0.5R \text{ and } r^F > 0.5R \end{cases} \tag{32}$$

Given ex-ante uncertainty about the returns from investment and the respective outside options, the expected payoff for each agent is the expectation of the payoff over the above three scenarios. To account for this uncertainty, say that the ex-ante probability (i.e. probability at date 0) that the outside option of the scientist is binding (i.e. $r^S > 0.5R$) is p^S. Similarly, say that the ex-ante probability that the outside option of the financier is binding (i.e. $r^F > 0.5R$) is p^F. Then, the probability that neither agent's outside option is binding is $1 - p^S - p^F$. Therefore the expected payoff of the scientist is

$$\begin{aligned} TS^S &= \left(1 - p^S - p^F\right) \cdot \frac{R}{2} + p^S\left(R - r^F\right) + p^F r^S \\ &= \left(1 + p^S - p^F\right) \cdot \frac{R}{2} - p^S r^F + p^F r^S \end{aligned} \tag{33}$$

where the expectation is taken at date 1 when the scientist decides the level of investment to make. Similarly, the expected payoff to the financier is

$$TS^F = \left(1 - p^S + p^F\right) \cdot \frac{R}{2} + p^S r^F - p^F r^S \tag{34}$$

Given these payoffs, the second-best investment levels e^{S*} and e^{F*} are given by the following first order conditions

$$\begin{aligned} \left(1 + p^S - p^F\right) \cdot \frac{R_S\left(e^{S*}, e^{F*}\right)}{2} + p^F \cdot r_S^S\left(e^{S*}, e^{F*}\right) - p^S \cdot r_S^F\left(e^{S*}, e^{F*}\right) &= 1 \\ \left(1 - p^S + p^F\right) \cdot \frac{R_F\left(e^{S*}, e^{F*}\right)}{2} + p^S \cdot r_F^F\left(e^{S*}, e^{F*}\right) - p^F \cdot r_F^S\left(e^{S*}, e^{F*}\right) &= 1 \end{aligned} \tag{35}$$

Comparing the above first order conditions to the those for the first-best level of investments, we can see that the scientist strictly overinvests (underinvests) when $p^F \cdot r_S^S\left(e^{S*}, e^{F*}\right) - p^S \cdot r_S^F\left(e^{S*}, e^{F*}\right) >< \frac{\left(1+p^S-p^F\right)}{2} \cdot R_S\left(e^{S*}, e^{F*}\right)$. Similarly, we can see that the financier strictly overinvests (underinvests) when $p^S \cdot r_F^F\left(e^{S*}, e^{F*}\right) - p^F \cdot r_F^S\left(e^{S*}, e^{F*}\right) >< \left(1 - p^S + p^F\right) \cdot \frac{R_F\left(e^{S*}, e^{F*}\right)}{2}$. Therefore, we get the problem of over-investment and under-investment as in the case of the 50:50 Nash bargaining solution.

The intuition for the generality of the results is the following. The over-investment and the under-investment result from the difference in the marginal values of the outside option and that of the surplus produced in the relationship. For the bargaining game used here, the no trade payoffs r^S and r^F do not affect the equilibrium payoffs over a certain range of the levels of the outside options. However, what is important for the analysis here is that the no-trade payoffs sometimes matter, not that they always matter. Therefore, with some amount of ex-ante uncertainty about the investment returns (i.e. surplus R and the no-trade payoffs), the no-trade payoffs will affect the equilibrium division of surplus with positive probabilities. Therefore, the analysis under alternative-offers bargaining is similar to the axiomatic 50:50 Nash bargaining.

A.2 Multi-tasking in investments

Here, we show that the results derived in the theory are unaltered if we allowed for multi-tasking in the investments. Since the additional notation makes the exposition very messy without providing any additional intuition, this part is presented as a separate appendix.

For notational convenience, we represent $R \equiv R^S + R^F$. Note that the separability here is in line with the separable technology assumed in the main model. Also, for notational convenience, we work with the general functions for the joint profit and the outside options rather than the explicit functional forms used in the main model.

Say, i^S denotes the investment by S to tailor her idea to the financier's complementary assets while e^S denotes investments by S to understand how to use the financier's assets. Similarly, let i^F denote investment by F to tailor its complementary asset to the scientist's idea while e^F denote investment by F to understand the scientist's idea. The joint profit is a function of each agent's investment to tailor the asset to that of the partner as well as by investments to understand how to use the asset of the partner. Thus

$$R\left(i^S, i^F, e^S, e^F, \alpha\right) \equiv R^S\left(i^S, e^S, \alpha\right) + R^F\left(i^F, e^F, \alpha\right) \quad (36)$$

In contrast, the outside option of each party is affected only by that party's investment to understand the partner's technology. Thus

$$r^S \equiv r^S\left(e^S, e^F, \alpha, \delta\right); r^F \equiv r^F\left(e^S, e^F, \alpha, \delta\right) \quad (37)$$

The technological assumptions remain similar to that in the main model:

$$\begin{array}{l} R_k^j > 0, R_{kk}^j < 0, R_{k\alpha}^j > 0, \quad j \in \{S, F\}, k \in \{i, e\} \\ r_e^j > 0, r_{ee}^j < 0, r_{e\alpha}^j > 0, \quad j \in \{S, F\} \\ r_e^S\left(\delta = 0\right) > r_e^S\left(\delta = 1\right); r_e^F\left(\delta = 0\right) < r_e^F\left(\delta = 1\right) \\ r_{e\theta}^F > 0, r_{e\theta}^S > 0 \end{array} \quad (38)$$

The net surplus generated in the relationship is given by

$$TS \equiv R^S\left(i^S, e^S, \alpha\right) + R^F\left(i^F, e^F, \alpha\right) - i^S - e^S - i^F - e^F \quad (39)$$

Therefore, the first order conditions for the first-best level of investments, which maximize TS, are given by

$$R_i^S\left(i^{SF}, e^{SF}, \alpha\right) = 1, \quad R_i^F\left(i^{FF}, e^{FF}, \alpha\right) = 1 \quad (40)$$

for the investment in tailoring each other's assets and

$$R_e^S\left(i^{SF}, e^{SF}, \alpha\right) = 1, \quad R_e^F\left(i^{FF}, e^{FF}, \alpha\right) = 1 \quad (41)$$

for the investment in understanding tailoring each other's assets.

Using $50:50$ Nash bargaining, the individual payoffs are given by

$$\begin{aligned} TS^S &= 0.5\left[R^S\left(i^S, e^S, \alpha\right) + R^F\left(i^F, e^F, \alpha\right) + r^S\left(e^S, e^F, \alpha\right) - r^F\left(e^S, e^F, \alpha\right)\right] - i^S - e^S \quad (42) \\ TS^F &= 0.5\left[R^S\left(i^S, e^S, \alpha\right) + R^F\left(i^F, e^F, \alpha\right) + r^F\left(e^S, e^F, \alpha\right) - r^S\left(e^S, e^F, \alpha\right)\right] - i^F - e^F \end{aligned}$$

Therefore, the first order conditions for the second-best level of investments in tailoring each other's assets are given by

$$R_i^S\left(i^{S*}, e^{S*}, \alpha\right) = 1, \quad R_i^F\left(i^{F*}, e^{F*}, \alpha\right) = 1 \tag{43}$$

while the first order conditions for the second-best level of investments in understanding each other's assets are given by

$$\begin{aligned} R_e^S\left(i^{S*}, e^{S*}, \alpha\right) + r_e^S\left(e^{S*}, e^{F*}, \alpha\right) - r_e^F\left(e^{S*}, e^{F*}, \alpha\right) &= 2 \\ R_e^F\left(i^{F*}, e^{F*}, \alpha\right) + r_e^F\left(e^{S*}, e^{F*}, \alpha\right) - r_e^S\left(e^{S*}, e^{F*}, \alpha\right) &= 2 \end{aligned} \tag{44}$$

Now, by comparing (40) and (43), it can be seen that $i^{j*} < i^{jF}$ for $j = S, F$ and for all α and δ. Therefore, both agents always underinvest in tailoring their assets to each other.

However, in the case of investments to understand each other's assets, both under- and over-investment are possible. If $r_e^F - r_e^S > R_e^F$, there is over-investment by the financier in understanding the idea. In other words, the financier invests in replicating the idea and in the process enhance his bargaining power. Similarly, if $r_e^S - r_e^F > R_e^S$, the scientist overinvests in understanding how to use the financier's complementary assets and thus enhance her bargaining power. In sum by examining the first-best and second-best investments, we can see that there is an optimal level of investment by the financier in understanding the scientist's idea so that he could develop the idea. Any additional investment by the financier is essentially to replicate the idea and thus enhance his bargaining power. Similar is the case with the innovator too.

Given the possibility of over- and under-investment in understanding the knowledge assets, all the results obtained in the main body of the paper get reproduced using this multi-tasking setup as well.

A.3 Asymmetric levels of access

We show that the symmetric effect of access on incentives is not a result of the reciprocity in access. To do so, we denote the access that the scientist receives from the financier to be α^S. Similarly, let the access that the financier recieves from the scientist be $\alpha^F \neq \alpha^S$. Now, since the scientist's (financier's) investments are affected by the access that the scientist (financier) receives, we can repeat the steps in Section 3.2 to obtain the following functional forms for the outside options:

$$\begin{aligned} r^S\left(e^S, e^F, \alpha^S, \alpha^F, \delta\right) &= a\left(0.5 + 0.5\mu - \delta\mu\right) + \left(1 - \delta\mu\right)\theta^S\phi\left(e^S, \alpha^S\right) - \gamma\left(1-\mu\right)\theta^F\phi\left(e^F, \alpha^F\right) \\ r^F\left(e^S, e^F, \alpha^S, \alpha^F, \delta\right) &= a\left(0.5 - 0.5\mu + \delta\mu\right) + \left(1 - \mu + \delta\mu\right)\theta^F\phi\left(e^F, \alpha^F\right) - \gamma\left(1-\mu\right)\theta^S\phi\left(e^S, \alpha^S\right) \end{aligned} \tag{45}$$

Given these outside options, the first-best and second-best choices of investments obtained by simplifying (16) and (17) are as follows:

$$\beta\phi_e\left(e^{SF}, \alpha^S\right) = 1, \quad \beta\phi_e\left(e^{FF}, \alpha^F\right) = 1 \tag{46}$$

and

$$\begin{aligned} 0.5\left[\beta + \left\{\left(1+\gamma\right)\left(1-\mu\right) + \left(1-\delta\right)\mu\right\}\theta^S\right]\phi_e\left(e^{S*}, \alpha^S\right) &= 1 \\ 0.5\left[\beta + \left\{\left(1+\gamma\right)\left(1-\mu\right) + \delta\mu\right\}\theta^F\right]\phi_e\left(e^{F*}, \alpha^F\right) &= 1 \end{aligned} \tag{47}$$

Given these first-order conditions, it follows that

$$\begin{aligned}\frac{de^{S*}}{d\alpha^F} &= \frac{de^{F*}}{d\alpha^S}=0 \\ \frac{de^{S*}}{d\alpha^S} &= \frac{de^{F*}}{d\alpha^F}=\frac{\phi_{e\alpha}}{|\phi_{ee}|}>0\end{aligned} \tag{48}$$

Thus, an increase in the access that the scientist (financier) receives enhances the scientist's (financier's) incentives but has *no effect* on the financier's (scientist's) investment.

$$TS^* = \beta\phi\left(e^{S*},\alpha^S\right)+\beta\phi\left(e^{F*},\alpha^F\right)-e^{S*}-e^{F*} \tag{49}$$

$$\begin{aligned}\frac{dTS^*}{d\alpha^S} &= \beta\phi_\alpha\left(e^{S*},\alpha^S\right)+\beta\phi_\alpha\left(e^{F*},\alpha^F\right) \\ &\quad +\left[\beta\phi_e\left(e^{S*},\alpha^S\right)-1\right]\frac{de^{S*}}{d\alpha^S}+\left[\beta\phi_e\left(e^{F*},\alpha^F\right)-1\right]\underbrace{\frac{de^{F*}}{d\alpha^S}}_{=0} \\ &= \beta\phi_\alpha\left(e^{S*},\alpha^S\right)+\beta\phi_\alpha\left(e^{F*},\alpha^F\right)+\left[\frac{\beta-\{(1+\gamma)(1-\mu)+(1-\delta)\mu\}\theta^S}{\beta+\{(1+\gamma)(1-\mu)+(1-\delta)\mu\}\theta^S}\right]\frac{\phi_{e\alpha}}{|\phi_{ee}|}\end{aligned} \tag{50}$$

Similarly

$$\frac{dTS^*}{d\alpha^F}=\beta\phi_\alpha\left(e^{S*},\alpha^S\right)+\beta\phi_\alpha\left(e^{F*},\alpha^F\right)+\left[\frac{\beta-\{(1+\gamma)(1-\mu)+\delta\mu\}\theta^F}{\beta+\{(1+\gamma)(1-\mu)+\delta\mu\}\theta^F}\right]\frac{\phi_{e\alpha}}{|\phi_{ee}|} \tag{51}$$

Thus, all the predictions obtained for reciprocal access generalize to the case of asymmetric access, with one change: θ^S only affects α^S and similarly θ^F only affects α^F. These proofs are not repeated in the interest of brevity.

Appendix B – Proofs of Propositions

Lemma: Consider a function $f \equiv f(x_1, x_2, \eta)$ where $x_1, x_2, \eta \in \mathbb{R}^+$ and η is a parameter. Define

$$\left(x_1^*(\eta), x_2^*(\eta)\right)=\underset{(x_1,x_2)}{\arg\max}\left[f(x_1,x_2,\eta)\right] \tag{52}$$

Assume f is smooth, possesses all second order derivatives and $f_{11}<0,\ f_{22}<0, f_{12}=f_{21}=0$ so that $\left(x_1^*(\eta), x_2^*(\eta)\right)$ as a interior maximum exists and is unique.

$$\begin{aligned}\frac{dx_1^*(\eta)}{d\eta} &= \frac{f_{1\eta}}{|f_{11}|},\frac{dx_2^*(\mu)}{d\mu}=\frac{f_{2\eta}}{|f_{22}|} \\ sign\left(\frac{dx_i^*(\eta)}{d\eta}\right) &= sign\left(f_{i\eta}\right)\ \ i\in 1,2\end{aligned} \tag{53}$$

Proof of Lemma: Since f is smooth and $\left(x_1^*(\eta), x_2^*(\eta)\right)$ is an interior solution, the first order conditions are

$$f_1\left(\left(x_1^*(\eta), x_2^*(\eta)\right)\right)=0 \text{ and } f_2\left(\left(x_1^*(\eta), x_2^*(\eta)\right)\right)=0 \tag{54}$$

Using implicit function theorem, there is a $(x_1^*(\eta), x_2^*(\eta))$ such that

$$\begin{aligned} f_{11}\frac{dx_1^*(\eta)}{d\eta}+f_{1\eta} &= 0 \Leftrightarrow \frac{dx_1^*(\eta)}{d\eta}=\frac{f_{1\eta}}{|f_{11}|} \\ f_{22}\frac{dx_2^*(\eta)}{d\eta}+f_{2\eta} &= 0 \Leftrightarrow \frac{dx_2^*(\mu)}{d\mu}=\frac{f_{2\eta}}{|f_{22}|} \end{aligned} \tag{55}$$

Therefore $sign\left(\frac{dx_i^*(\eta)}{d\eta}\right)=sign(f_{i\eta})$ for $i\in 1,2.\diamondsuit$

Proof of Lemma 1:

$$\begin{aligned} r^S+r^F &= a+\theta^S\phi\left(e^S,\alpha\right)[1-\delta\mu-\gamma(1-\mu)]+\theta^F\phi\left(e^F,\alpha\right)[1-\mu(1-\delta)-\gamma(1-\mu)] \\ &< a+\phi\left(e^F,\alpha\right)+\phi\left(e^S,\alpha\right)<a+\beta\phi\left(e^F,\alpha\right)+\beta\phi\left(e^S,\alpha\right)=R \end{aligned} \tag{56}$$

since $\delta=0$ or $1, 0\le\mu\le1, 0\le\gamma\le1, 0\le\theta^S,\theta^F\le1$ and $\beta>1$. Therefore

$$r^S+r^F<R\ \forall e^S,e^F,\theta^S,\theta^F,\alpha,\delta \tag{57}$$

$$\begin{aligned} & r^S(Physical)+r^F(Physical) \\ =\ & a+\theta^F\phi\left(e^F,\alpha\right)[1-(1-\delta)\{1-\beta_P(1-\gamma)\}]+\theta^S\phi\left(e^S,\alpha\right)(1-\delta)(\beta_P-\gamma) \\ <\ & a+\phi\left(e^F,\alpha\right)+\phi\left(e^S,\alpha\right)<a+\beta\phi\left(e^F,\alpha\right)+\beta\phi\left(e^S,\alpha\right)=R \end{aligned} \tag{58}$$

since $\delta=0$ or $1, 0\le\mu\le1, 0\le\gamma\le1, 0\le\theta^S,\theta^F\le1, 0\le\beta_P\le1$ and $\beta>1$. Therefore

$$r^S(Physical)+r^F(Physical)<R\forall e^S,e^F,\alpha,\delta \tag{59}$$

$\diamondsuit$

Proof of Propositions 1 and 2: follow directly from the functional form for r^S and r^F (equations (8)).$\diamondsuit$

Proof of Proposition 3: The first-best investments $\left(e^{SF},e^{FF}\right)$ are given by the first order conditions (16) while the second-best investments $\left(e^{S*},e^{F*}\right)$ are given by the first order conditions (17). Define

$$f(x_1,x_2,\lambda)=(1-\lambda)R(x_1,x_2)+\lambda r^S(x_1,1-x_2)+\lambda r^F(1-x_1,x_2)-x_1-x_2 \tag{60}$$

Then it follows that $\left(e^{SF},e^{FF}\right)=(x_1^*(0),x_2^*(0))$ while $\left(e^{A*},e^{B*}\right)=(x_1^*(0.5),x_2^*(0.5))$. Note that $f_{11}<0, f_{22}<0$ and $f_{12}=0$ are satisfied in this case. Further, $f_{13}=r_S^S-r_S^F-R_S=2\left(\Pi_S^S-TS_S\right)$ and $f_{23}=r_F^F-r_F^S-R_F=2\left(\Pi_F^F-TS_F\right)$ using 15. The results then follow by applying the Lemma. $\diamondsuit$

Proof of Proposition 4: The first-order conditions for the equilibrium investments $\left(e^{S*},e^{F*}\right)$ in the case of physical assets are given by equations (20) while the first-best investments are given by (16). Define

$$\begin{aligned} f(x_1,x_2,\eta) &= (1-\eta)\beta[\phi(x_1)+\phi(x_2)]+\eta[\beta+(\gamma+\beta_P)(1-\delta)]\phi(x_1) \\ &\quad +\eta[\beta+\beta_P(1+\gamma)(1-\delta)+\delta]\phi(x_2)-x_1-x_2 \end{aligned} \tag{61}$$

Then it follows that $\left(e^{SF},e^{FF}\right)=(x_1^*(0),x_2^*(0))$ while $\left(e^{S*},e^{F*}\right)=(x_1^*(0.5),x_2^*(0.5))$. Also in this case, $f_{11}<0,\ f_{22}<0, f_{21}=0$. Using the Lemma, it therefore follows that $(\gamma+\beta_P)(1-\delta)\theta^S<\beta$

$\Leftrightarrow e^{S*} < e^{SF}$ and $\{(1+\gamma)(1-\delta)\beta_P + \delta\}\theta^S < \beta \Leftrightarrow e^{F*} < e^{FF}$. We now show that these two conditions are satisfied.

$(\gamma + \beta_P)(1-\delta)\theta^S \leq 1 + \beta_P < \beta$ since $0 \leq \theta^S \leq 1, 0 \leq \gamma \leq 1$ and $\delta = 0, 1$ and $\beta_P < \beta - 1$ from (10). Also, $\beta_P < \beta - 1 < 0.5\beta$ using (10) and (2). Since $0 < \gamma < 1$ and $0 < \theta^F < 1$, it follows that $[(1+\gamma)(1-\delta)\beta_P + \delta]\theta^F < (1-\delta)\beta + \delta = \beta - (\beta-1)\delta < \beta$ since $1 < \beta < 2$.

Now

$$TS = R\left(e^{F*}, e^{S*}, \alpha\right) - e^{F*} - e^{S*} \tag{62}$$
$$\frac{dTS}{d\alpha} = R_\alpha + \left[R_F\left(e^{F*}, e^{S*}, \alpha\right) - 1\right]\frac{de^{F*}}{d\alpha} + \left[R_S\left(e^{F*}, e^{S*}, \alpha\right) - 1\right]\frac{de^{S*}}{d\alpha}$$

Now using the first order conditions (20),

$$R_F\left(e^{F*}, e^{S*}, \alpha\right) - 1 = \beta\phi_e\left(e^{F*}\right) - 1 = \frac{2\beta}{\beta + (1+\gamma)(1-\delta)\beta_P + \delta} - 1 > 0 \tag{63}$$
$$R_S\left(e^{F*}, e^{S*}, \alpha\right) - 1 = \beta\phi_e\left(e^{S*}\right) - 1 = \frac{2\beta}{\beta + (\gamma + \beta_P)(1-\delta)} - 1 > 0$$

Since $\frac{de^{i*}}{d\alpha} = \frac{\phi_{e\alpha}}{|\phi_{ee}|} > 0, i = S, F$ using the first order conditions (22), it follows that $\frac{dTS}{d\alpha} > 0 \Rightarrow \alpha^* = 1.\diamondsuit$

Proof of Proposition 5: The first-best investments $\left(e^{FF}, e^{SF}\right)$ are given by the first order conditions (16) while the second-best investments $\left(e^{F*}, e^{S*}\right)$ are given by the first order conditions (22). Define

$$\begin{aligned} f(x_1, x_2, \eta) &= (1-\eta)\beta[\phi(x_1) + \phi(x_2)] + \eta[(1+\gamma)(1-\mu) + \delta\mu]\phi(x_1) \\ &\quad + \eta[(1+\gamma)(1-\mu) + (1-\delta)\mu]\phi(x_2) - x_1 - x_2 \end{aligned} \tag{64}$$

Then it follows that $\left(e^{FF}, e^{SF}\right) = (x_1^*(0), x_2^*(0))$ while $\left(e^{F*}, e^{S*}\right) = (x_1^*(0.5), x_2^*(0.5))$. Also in this case, $f_{11} < 0$, $f_{22} < 0, f_{21} = 0$.

$$\begin{aligned} f_{1\eta} &= \left[-\beta + \{1 - \mu + \delta\mu + \gamma(1-\mu)\}\theta^F\right]\phi_F\left(e^F\right) \\ f_{2\eta} &= \left[-\beta + \{1 - \delta\mu + \gamma(1-\mu)\}\theta^S\right]\phi_S\left(e^S\right) \end{aligned} \tag{65}$$

The result follows from using Lemma.$\diamondsuit$

Proof of Corollary 1: Differentiating the first order conditions (22) wrt γ, μ, θ^i and β, we obtain

$$\frac{de^{i*}}{d\gamma} = \frac{2(1-\mu)\theta^i}{|\phi_{ee}|\left[\beta + \{(1+\gamma)(1-\mu) + \Delta^i\mu\}\theta^i\right]^2} > 0 \tag{66}$$

$$\frac{de^{i*}}{d\mu} = \frac{2\left(\Delta^i - 1 - \gamma\right)\theta^i}{|\phi_{ee}|\left[\beta + \{(1+\gamma)(1-\mu) + \Delta^i\mu\}\theta^i\right]^2} < 0 \because \Delta^i \leq 1$$

$$\frac{de^{i*}}{d\theta^i} = \frac{2\left\{(1+\gamma)(1-\mu) + \Delta^i\mu\right\}}{|\phi_{ee}|\left[\beta + \{(1+\gamma)(1-\mu) + \Delta^i\mu\}\theta^i\right]^2} > 0$$

$$\frac{de^{i*}}{d\beta} = \frac{2}{|\phi_{ee}|\left[\beta + \{(1+\gamma)(1-\mu) + \Delta^i\mu\}\theta^i\right]^2} > 0$$

$$\frac{|\phi_{ee}|}{2\theta^i}\frac{d^2e^{i*}}{d\gamma d\mu} = \frac{-\left[\beta + \left\{(1+\gamma)(1-\mu) + \Delta^i\mu\right\}\theta^i\right] - 2(1-\mu)\theta^i\left[1 - \Delta^i + \gamma\right]}{\left[\beta + \{(1+\gamma)(1-\mu) + \Delta^i\mu\}\theta^i\right]^3} < 0 \tag{67}$$

Differentiating the first order conditions (16) wrt γ, μ, θ^i and β, we obtain

$$\frac{de^{iF}}{d\beta} = \frac{2}{|\phi_{ee}|} > \frac{de^{i*}}{d\beta}; \frac{de^{iF}}{d\gamma} = \frac{de^{iF}}{d\mu} = \frac{de^{iF}}{d\theta^i} = 0. \tag{68}$$

which completes the proof.◊

Proof of Proposition 6: (i) Differentiate (22) wrt α we get for $i = F, S$

$$\phi_{i\alpha} + \phi_{ii}\frac{de^{i*}}{d\alpha} = 0 \Leftrightarrow \frac{de^{i*}}{d\alpha} = -\frac{\phi_{i\alpha}}{\phi_{ii}} = \frac{\phi_{i\alpha}}{|\phi_{ee}|} > 0 \tag{69}$$

(ii) Differentiate (22) wrt δ we get

$$\frac{de^{S*}}{d\delta} = -\frac{2\theta^S\mu}{|\phi_{ee}|\left[\beta + \{(1+\gamma)(1-\mu) + (1-\delta)\mu\}\theta^S\right]^2} < 0 \Leftrightarrow e^{S*}(\delta = 1) < e^{S*}(\delta = 0) \tag{70}$$

$$\frac{de^{F*}}{d\delta} = \frac{2\theta^F\mu}{|\phi_{ee}|\left[\beta + \{(1+\gamma)(1-\mu) + \delta\mu\}\theta^F\right]^2} > 0 \Leftrightarrow e^{F*}(\delta = 1) > e^{F*}(\delta = 0)$$

which completes the proof.◊

Proof of Corollary 2: follows directly from combining Propositions 5 and 6.◊

Proof of Corollary 3: From the first-order conditions (22), it follows that if $\mu = 0$, then for $i \in S, F$

$$\phi_i\left(e^{i*}(\delta = 1)\right) = \phi_i\left(e^{i*}(\delta = 0)\right) \Leftrightarrow e^{i*}(\delta = 1) = e^{i*}(\delta = 0) \tag{71}$$

which completes the proof.◊

Proof of Corollary 4: From the first-order conditions (22), it follows that if $\theta^S = \theta^F$, then

$$\phi_S\left(e^{S*}(\delta = 1)\right) = \phi_F\left(e^{F*}(\delta = 0)\right) \Leftrightarrow e^{S*}(\delta = 1) = e^{F*}(\delta = 0) \tag{72}$$

$$\phi_S\left(e^{S*}(\delta = 0)\right) = \phi_F\left(e^{F*}(\delta = 1)\right) \Leftrightarrow e^{S*}(\delta = 0) = e^{F*}(\delta = 1)$$

Therefore, the result follows.$\diamondsuit$

Proof of Proposition 7:

$$TS^* = R\left(e^{F*}, e^{S*}, \alpha\right) - e^{F*} - e^{S*} \tag{73}$$

Now using the first order conditions (22) and using (69)

$$\frac{dTS^*}{d\alpha} = \tag{74}$$
$$\beta\left[\phi_\alpha\left(e^{S*}\right) + \phi_\alpha\left(e^{F*}\right)\right] + \left[\frac{2\beta}{\beta + \left[\delta\mu + (1+\gamma)(1-\mu)\right]\theta^F} + \frac{2\beta}{\beta + \left[(1-\delta)\mu + (1+\gamma)(1-\mu)\right]\theta^S} - 2\right]\frac{\phi_{e\alpha}}{|\phi_{ee}|}$$
$$\frac{d^2TS^*}{d\alpha^2} = 2\beta\left[\phi_{\alpha\alpha} + \frac{\phi_{e\alpha}^2}{|\phi_{ee}|}\right] < 0 \because \phi_{\alpha\alpha} < 0, \phi_{\alpha\alpha}\phi_{ee} - \phi_{e\alpha}^2 > 0, \beta > 1$$

Therefore, an interior maximum exists for α. Using Lemma and $\frac{d^2TS}{d\alpha^2}$, it follows that

$$sign\left(\frac{d\alpha^*}{d\lambda}\right) = sign\left(\frac{d^2TS^*}{d\alpha d\lambda}\right) \tag{75}$$

Further note that since $\frac{d^2TS^*}{d\alpha^2}$ is independent of the parameters γ and μ, it follows that the above is valid for higher order comparative statics as well.

Define

$$\underline{\gamma} = \beta\left(\frac{1+\beta\phi_\alpha^{\min}}{1-\beta\phi_\alpha^{\min}}\right) - 1 \tag{76}$$

Then it follows that $\gamma < \underline{\gamma} \Rightarrow (1+\gamma) < \beta\left[\frac{2}{1-\beta\phi_\alpha^{\min}} - 1\right] < \beta\left[\frac{2}{1-\beta\phi_\alpha} - 1\right]$. Therefore, $\beta + (1+\gamma) < \frac{2\beta}{1-\beta\phi_\alpha}$.

Since $0 \le \theta^F, \theta^S \le 1$ and $\delta = 0, 1$, it follows that

$$\begin{aligned} \beta + \left[\delta\mu + (1+\gamma)(1-\mu)\right]\theta^F &< \beta + \mu + (1+\gamma)(1-\mu) < \beta + 1 + \gamma < \frac{2\beta}{1-\beta\phi_\alpha} \\ \beta + \left[(1-\delta)\mu + (1+\gamma)(1-\mu)\right]\theta^F &< \beta + \mu + (1+\gamma)(1-\mu) < \beta + 1 + \gamma < \frac{2\beta}{1-\beta\phi_\alpha} \end{aligned} \tag{77}$$

Therefore, $\gamma < \underline{\gamma} \Rightarrow \frac{dTS}{d\alpha} > 0 \Rightarrow \alpha^* = 1$. If $\gamma > \underline{\gamma}$, then interior solutions for α exist.

Differentiating (74) wrt γ we get

$$\frac{|\phi_{ee}|}{\phi_{e\alpha}}\frac{d^2TS^*}{d\alpha d\gamma} = \tag{78}$$
$$\frac{-2\beta\theta^F(1-\mu)}{\left\{\beta + \left[\delta\mu + (1+\gamma)(1-\mu)\right]\theta^F\right\}^2} - \frac{2\beta\theta^S(1-\mu)}{\left\{\beta + \left[(1-\delta)\mu + (1+\gamma)(1-\mu)\right]\theta^S\right\}^2} < 0$$
$$\frac{d\alpha^*}{d\gamma} \le 0 \text{ allowing for boundary solutions.}$$

Differentiating expression (74) wrt μ we get

$$
\begin{aligned}
&\frac{|\phi_{ee}|}{2\beta\phi_{e\alpha}}\frac{d^2TS^*}{d\alpha d\mu} && (79)\\
= \quad & \frac{(1-\delta+\gamma)\,\theta^F}{\left\{\beta+[\delta\mu+(1+\gamma)(1-\mu)]\,\theta^F\right\}^2}+\frac{(2+\gamma-\delta)\,\theta^S}{\left\{\beta+[(1-\delta)\,\mu+(1+\gamma)(1-\mu)]\,\theta^S\right\}^2}>0\\
\Leftrightarrow \quad & \frac{d\alpha^*}{d\mu}\geq 0 \text{ allowing for boundary solutions.}
\end{aligned}
$$

Differentiating expression (79) wrt γ we get

$$
\frac{|\phi_{ee}|}{2\beta\phi_{e\alpha}}\frac{d^3TS^*}{d\alpha d\mu d\gamma}=\frac{\beta+[\delta\mu+(2\delta-1-\gamma)(1-\mu)]\,\theta^F}{\left\{\beta+[\delta\mu+(1+\gamma)(1-\mu)]\,\theta^F\right\}^3}+\frac{\beta+[(1-\delta)\,\mu+(1-2\delta-\gamma)(1-\mu)]\,\theta^S}{\left\{\beta+[(1-\delta)\,\mu+(1+\gamma)(1-\mu)]\,\theta^S\right\}^2}
$$

If $\delta=0$, then the RHS equals

$$
\frac{\beta-(1+\gamma)(1-\mu)\,\theta^F}{\left\{\beta+(1+\gamma)(1-\mu)\,\theta^F\right\}^3}+\frac{\beta+(1-\gamma+\gamma\mu)\,\theta^S}{\left\{\beta+(1+\gamma-\gamma\mu)\,\theta^S\right\}^2} \tag{80}
$$

If $\delta=1$, then the RHS equals

$$
\frac{\beta-(1+\gamma)(1-\mu)\,\theta^S}{\left\{\beta+(1+\gamma)(1-\mu)\,\theta^S\right\}^3}+\frac{\beta+(1-\gamma+\gamma\mu)\,\theta^F}{\left\{\beta+(1+\gamma-\gamma\mu)\,\theta^F\right\}^2} \tag{81}
$$

In each of the above two expressions, term 2 is always positive since $1-\gamma(1-\mu)>0$ while term 1 could be negative. The minimum value of term 1 is obtained at the boundary value $\gamma=2,\mu=0$ since it is easy to check that term 1 is monotonically increasing (decreasing) in $\mu\,(\gamma)$. Therefore

$$
\frac{|\phi_{ee}|}{2\beta\phi_{e\alpha}}\frac{d^3TS^*}{d\alpha d\mu d\gamma}>\frac{\beta}{(\beta+2)^3}+\frac{\beta-2}{(\beta+2)^3}=\frac{2\beta-2}{(\beta+2)^3}>0\because\beta>1\Leftrightarrow\frac{d^2\alpha^*}{d\mu d\gamma}\geq 0 \tag{82}
$$

allowing for boundary solutions.$\diamondsuit$

Proof of Proposition 8: Differentiating (74) wrt β respectively, we obtain

$$
\begin{aligned}
&\frac{1}{2}\frac{d^2TS^*}{d\alpha d\beta}=\phi_\alpha+ && (83)\\
&\left[\frac{[\delta\mu+(1+\gamma)(1-\mu)]\,\theta^F}{\left\{\beta+[\delta\mu+(1+\gamma)(1-\mu)]\,\theta^F\right\}^2}+\frac{[(1-\delta)\,\mu+(1+\gamma)(1-\mu)]\,\theta^S}{\left\{\beta+[(1-\delta)\,\mu+(1+\gamma)(1-\mu)]\,\theta^S\right\}^2}\right]\frac{\phi_{e\alpha}}{|\phi_{ee}|}>0\\
&\frac{d\alpha^*}{d\beta}\geq 0
\end{aligned}
$$

allowing for boundary solutions.$\diamondsuit$

Proof of Proposition 9: Differentiating (74) wrt θ^S and θ^F respectively, we obtain

$$\begin{aligned}\frac{|\phi_{ee}|}{2\beta\phi_{e\alpha}}\frac{d^2TS^*}{d\alpha d\theta^F} &= -\frac{\{\delta\mu+(1+\gamma)(1-\mu)\}}{\left[\beta+\{\delta\mu+(1+\gamma)(1-\mu)\}\theta^F\right]^2}<0\Leftrightarrow\frac{d\alpha^*}{d\theta^F}\leq 0 \\ \frac{|\phi_{ee}|}{2\beta\phi_{e\alpha}}\frac{d^2TS^*}{d\alpha d\theta^S} &= -\frac{\{(1-\delta)\mu+(1+\gamma)(1-\mu)\}}{\left[\beta+\{(1-\delta)\mu+(1+\gamma)(1-\mu)\}\theta^S\right]^2}<0\Leftrightarrow\frac{d\alpha^*}{d\theta^S}\leq 0\end{aligned} \tag{84}$$

allowing for boundary solutions.♢

Proofs of Propositions 10 and 11:

$$TS^*(\delta)=\sum_{i\in S,F}\left[\beta\phi\left(e^{i*}(\delta)\right)-e^{i*}(\delta)\right] \tag{85}$$

Using Corollaries 3 and 4, it follows that if either $\theta^S=\theta^F$ or $\mu=0\Rightarrow$

$$\begin{aligned}& TS^*(\delta=1)-TS^*(\delta=0) \\ =& \sum_{i\in S,F}\left[\beta\phi\left(e^{i*}(\delta=1)\right)-e^{i*}(\delta=1)\right]-\sum_{i\in S,F}\left[\beta\phi\left(e^{i*}(\delta=0)\right)-e^{i*}(\delta=0)\right] \\ =& \; 0\end{aligned} \tag{86}$$

which completes the proof.♢

Proof of Proposition 12: Using the expression (74) for $\frac{dTS}{d\alpha}$

$$\frac{|\phi_{ee}|}{2\beta\phi_{e\alpha}}\frac{dTS}{d\alpha}=\frac{\phi_\alpha|\phi_{ee}|}{\phi_{e\alpha}}-\frac{1}{\beta}+\frac{1}{\beta+\{\delta\mu+(1+\gamma)(1-\mu)\}\theta^F}+\frac{1}{\beta+\{(1-\delta)\mu+(1+\gamma)(1-\mu)\}\theta^S} \tag{87}$$

Define $\underline{\lambda}=\frac{\phi_\alpha^{\min}|\phi_{ee}|}{\phi_{e\alpha}}$ and $\overline{\lambda}=\frac{\phi_\alpha^{\max}|\phi_{ee}|}{\phi_{e\alpha}}$. Therefore,

$$\begin{aligned}\frac{1}{\beta+\{\delta\mu+(1+\gamma)(1-\mu)\}\theta^F}+\frac{1}{\beta+\{(1-\delta)\mu+(1+\gamma)(1-\mu)\}\theta^S} &> \frac{1}{\beta}-\underline{\lambda}\Leftrightarrow\alpha^*=1 \\ \frac{1}{\beta+\{\delta\mu+(1+\gamma)(1-\mu)\}\theta^F}+\frac{1}{\beta+\{(1-\delta)\mu+(1+\gamma)(1-\mu)\}\theta^S} &< \frac{1}{\beta}-\overline{\lambda}\Leftrightarrow\alpha^*=0\end{aligned} \tag{88}$$

else $0<\alpha^*<1$,which completes the proof.♢

Proof of Proposition 13: From Proposition 9, $TS^*(\delta=1)-TS^*(\delta=0)=0$ if $\theta^S=\theta^F=0$. Using mean value theorem, it follows that

$$TS^*(\delta=1)-TS^*(\delta=0)=\sum_{i\in S,F}\frac{\partial\left[TS^*(\delta=1)-TS^*(\delta=0)\right]}{\partial\theta^i}\theta^i \tag{89}$$

$$\begin{aligned}& \frac{\partial\left(TS^*(\delta=1)-TS^*(\delta=0)\right)}{\partial\theta^i} \\ =& \sum_{i\in S,F}\left[\beta\phi_e\left(e^{i*}(\delta=1)\right)-1\right]\frac{de^{i*}(\delta=1)}{d\theta^i}-\sum_{i\in S,F}\left[\beta\phi_e\left(e^{i*}(\delta=0)\right)-1\right]\frac{de^{i*}(\delta=0)}{d\theta^i}\end{aligned} \tag{90}$$

$$\frac{|\phi_{ee}|}{2}\left[TS^*(\delta=1)-TS^*(\delta=0)\right] \tag{91}$$

$$= \frac{\left(\beta-(1+\gamma)(1-\mu)\theta^S\right)(1+\gamma)(1-\mu)\theta^S}{\left[\beta+(1+\gamma)(1-\mu)\theta^S\right]^3}-\frac{\left(\beta-[1+\gamma(1-\mu)]\theta^S\right)[1+\gamma(1-\mu)]\theta^S}{\left[\beta+[1+\gamma(1-\mu)]\theta^S\right]^3}$$
$$-\frac{\left(\beta-(1+\gamma)(1-\mu)\theta^F\right)(1+\gamma)(1-\mu)\theta^F}{\left[\beta+(1+\gamma)(1-\mu)\theta^F\right]^3}+\frac{\left(\beta-[1+\gamma(1-\mu)]\theta^F\right)[1+\gamma(1-\mu)]\theta^F}{\left[\beta+[1+\gamma(1-\mu)]\theta^F\right]^3}$$

Let

$$G(\theta)=\frac{(\beta-(1+\gamma)(1-\mu)\theta)(1+\gamma)(1-\mu)\theta}{[\beta+(1+\gamma)(1-\mu)\theta]^3}-\frac{(\beta-[1+\gamma(1-\mu)]\theta)[1+\gamma(1-\mu)]\theta}{[\beta+[1+\gamma(1-\mu)]\theta]^3} \tag{92}$$

Using a Taylor series expansion and retaining first and second order terms, we get

$$G(\theta)\approx\frac{\mu}{\beta^3}\left[4(2+2\gamma-2\gamma\mu-\mu)\theta^2-\beta\theta\right] \tag{93}$$

Therefore,

$$\frac{|\phi_{ee}|\beta^3}{2\mu}\left[TS^*(\delta=1)-TS^*(\delta=0)\right]=\left(\theta^S-\theta^F\right)\left[4(2+2\gamma-2\gamma\mu-\mu)\left(\theta^S+\theta^F\right)-\beta\right] \tag{94}$$

(i) If $\theta^S+\theta^F\leq\frac{0.25\beta}{2+2\gamma-2\gamma\mu-\mu}$ then $\theta^S\gtreqless\theta^F\Rightarrow TS^*(\delta=1)\lesseqgtr TS^*(\delta=0)$. (ii) If $\theta^S+\theta^F>\frac{0.25\beta}{2+2\gamma-2\gamma\mu-\mu}$ then $\theta^S\gtreqless\theta^F\Rightarrow TS^*(\delta=1)\gtreqless TS^*(\delta=0)$.$\diamondsuit$

Proof of Proposition 14: Given our focus on $\omega^{FB}\in(\underline{\omega},\overline{\omega})$, it follows that

$$\left.\frac{dTS^*}{d\omega}\right|_{\omega=\omega^{FB}}=0 \tag{95}$$

Using expressions for $TS^*,\Pi^{S*}(\omega)$ and $\Pi^{F*}(\omega)$ (see equations (24)), we get

$$\frac{d^2TS^*}{d\alpha^2} = 2\beta\left[\phi_{\alpha\alpha}+\frac{\phi_{e\alpha}^2}{|\phi_{ee}|}\right]<0\because\phi_{\alpha\alpha}<0,\phi_{\alpha\alpha}\phi_{ee}-\phi_{e\alpha}^2>0,\beta>1 \tag{96}$$
$$\frac{d^2\Pi^{S*}}{d\alpha^2} = \left[\beta+0.5\{(1+\gamma)(1-\mu)+(1-\delta)\mu\}\theta^S-0.5\{(1+\gamma)(1-\mu)+\delta\mu\}\theta^F\right]\left[\phi_{\alpha\alpha}+\frac{\phi_{e\alpha}^2}{|\phi_{ee}|}\right]<0$$
$$\frac{d^2\Pi^{F*}}{d\alpha^2} = \left[\beta-0.5\{(1+\gamma)(1-\mu)+(1-\delta)\mu\}\theta^S+0.5\{(1+\gamma)(1-\mu)+\delta\mu\}\theta^F\right]\left[\phi_{\alpha\alpha}+\frac{\phi_{e\alpha}^2}{|\phi_{ee}|}\right]<0$$

since $\{(1+\gamma)(1-\mu)+\delta\mu\}\theta^F\leq 2,\{(1+\gamma)(1-\mu)+(1-\delta)\mu\}\theta^S\leq 2$ and $\beta>1$.

Now since

$$TS^*=\Pi^{S*}+\Pi^{F*}\Leftrightarrow\left.\frac{d\Pi^{S*}}{d\omega}\right|_{\omega=\omega^{FB}}=-\left.\frac{d\Pi^{F*}}{d\omega}\right|_{\omega=\omega^{FB}} \tag{97}$$

Case 1: $\left.\frac{d\Pi^{S*}}{d\omega}\right|_{\omega=\omega^{FB}}\leq 0\Leftrightarrow\left.\frac{d\Pi^{F*}}{d\omega}\right|_{\omega=\omega^{FB}}\geq 0$. Now since $\left.\frac{d\Pi^{S*}}{d\omega}\right|_{\omega=\omega^S}=\left.\frac{d\Pi^{F*}}{d\omega}\right|_{\omega=\omega^F}=0\Leftrightarrow\left.\frac{d\Pi^{S*}}{d\omega}\right|_{\omega=\omega^{FB}}\leq\left.\frac{d\Pi^{S*}}{d\omega}\right|_{\omega=\omega^S}\Leftrightarrow\omega^{FB}\geq\omega^S$ using $\frac{d^2\Pi^{S*}}{d\omega^2}<0$. Similarly, $\left.\frac{d\Pi^{F*}}{d\omega}\right|_{\omega=\omega^{FB}}\geq\left.\frac{d\Pi^{F*}}{d\omega}\right|_{\omega=\omega^F}\Leftrightarrow\omega^{FB}\leq\omega^F$ using $\frac{d^2\Pi^{F*}}{d\omega^2}<0$.

Case 2: $\left.\frac{d\Pi^{S*}}{d\omega}\right|_{\omega=\omega^{FB}} > 0 \Leftrightarrow \left.\frac{d\Pi^{F*}}{d\omega}\right|_{\omega=\omega^{FB}} < 0.$ $\Leftrightarrow \omega^F > \omega^{FB} > \omega^S$ using similar arguments to above.$\diamondsuit$

Proof of Proposition 15: Given α, δ, Ω and $\omega \in \left\{\omega^{FB}, \omega^S, \omega^F\right\}$, the first-best investments $\left(e^{FF}, e^{SF}\right)$ are given by the first order conditions

$$\beta\phi_e\left(e^{iF}\left(\omega\left(\alpha,\Omega\right),\alpha\right),\omega\left(\alpha,\Omega\right),\alpha\right)=1 \tag{98}$$

while given α, δ, Ω and $\omega \in \left\{\omega^{FB}, \omega^S, \omega^F\right\}$, the second-best investments $\left(e^{F*}, e^{S*}\right)$ are given by the first order conditions

$$0.5\left[\beta+\left\{(1+\gamma)(1-\mu)+\Delta^i\mu\right\}\theta^i\right]\phi_e\left(e^{i*}\left(\omega\left(\alpha,\Omega\right),\alpha,\delta\right),\omega\left(\alpha,\Omega\right),\alpha\right)=1 \tag{99}$$

Define

$$\begin{aligned} f\left(x_1,x_2,\eta\right) &= (1-\eta)\beta\left[\phi\left(x_1\right)+\phi\left(x_2\right)\right]+\eta\left[(1+\gamma)(1-\mu)+\delta\mu\right]\phi\left(x_1\right) \\ &\quad +\eta\left[(1+\gamma)(1-\mu)+(1-\delta)\mu\right]\phi\left(x_2\right)-x_1-x_2 \end{aligned} \tag{100}$$

Then it follows that $\left(e^{FF}, e^{SF}\right) = \left(x_1^*(0), x_2^*(0)\right)$ while $\left(e^{F*}, e^{S*}\right) = \left(x_1^*(0.5), x_2^*(0.5)\right)$. Also in this case, $f_{11} < 0$, $f_{22} < 0$, $f_{21} = 0$.

$$\begin{aligned} f_{1\eta} &= \left[-\beta+\left\{1-\mu+\delta\mu+\gamma(1-\mu)\right\}\theta^F\right]\phi_F\left(e^F\right) \\ f_{2\eta} &= \left[-\beta+\left\{1-\delta\mu+\gamma(1-\mu)\right\}\theta^S\right]\phi_S\left(e^S\right) \end{aligned} \tag{101}$$

Therefore, the conditions for over- and under-investment are as in Proposition 5, with the only difference that these conditions are given α, δ, Ω and $\omega \in \left\{\omega^{FB}, \omega^S, \omega^F\right\}$ instead of given α, δ.

Differentiating (99) w.r.t. ω we get

$$\phi_{ee}\frac{de^{i*}}{d\omega}+\phi_{e\omega}=0 \Leftrightarrow \frac{de^{i*}}{d\omega}=\frac{\phi_{e\omega}}{\left|\phi_{ee}\right|}>0 \because \phi_{e\omega}>0 \tag{102}$$

The first order conditions for $\omega^{FB}, \omega^S, \omega^F$ are given by

$$\begin{aligned} &\left[\beta\phi_\omega\left(e^{S*},\omega^{FB},\alpha\right)+\left\{\beta\phi_e\left(e^{S*},\omega^{FB},\alpha\right)-1\right\}\frac{\phi_{e\omega}}{\left|\phi_{ee}\right|}\right] \\ &+\left[\beta\phi_\omega\left(e^{F*},\omega^{FB},\alpha\right)+\left\{\beta\phi_e\left(e^{F*},\omega^{FB},\alpha\right)-1\right\}\frac{\phi_{e\omega}}{\left|\phi_{ee}\right|}\right]=0 \end{aligned} \tag{103}$$

$$\begin{aligned} &\left[\beta+\left\{(1+\gamma)(1-\mu)+(1-\delta)\mu\right\}\theta^S\right]\phi_\omega\left(e^{S*},\omega^S,\alpha\right) \\ &+\left[\beta-\left\{(1+\gamma)(1-\mu)+\delta\mu\right\}\theta^F\right]\left[\phi_\omega\left(e^{F*},\omega^S,\alpha\right)+\phi_e\left(e^{F*},\omega^S,\alpha\right)\frac{\phi_{e\omega}}{\left|\phi_{ee}\right|}\right]=0 \end{aligned} \tag{104}$$

$$\begin{aligned} &\left[\beta+\left\{(1+\gamma)(1-\mu)+\delta\mu\right\}\theta^F\right]\phi_\omega\left(e^{F*},\omega^F,\alpha\right) \\ &+\left[\beta-\left\{(1+\gamma)(1-\mu)+(1-\delta)\mu\right\}\theta^S\right]\left[\phi_\omega\left(e^{S*},\omega,\alpha\right)+\phi_e\left(e^{S*},\omega,\alpha\right)\frac{\phi_{e\omega}}{\left|\phi_{ee}\right|}\right]=0 \end{aligned} \tag{105}$$

Differentiating (103) w.r.t. α and simplifying we get

$$\left[\left|\phi_{ee}\right|\phi_{\omega\omega}+\left(\phi_{e\omega}\right)^2\right]\frac{d\omega^{FB}}{d\alpha}+\phi_{\omega\alpha}\left|\phi_{ee}\right|+\phi_{e\omega}\phi_{e\alpha}=0 \tag{106}$$

Given the assumption of concavity, we have $\phi_{ee}\phi_{\omega\omega} - (\phi_{e\omega})^2 > 0$. Therefore,

$$\frac{d\omega^{FB}}{d\alpha} = \frac{\phi_{\omega\alpha}|\phi_{ee}| + \phi_{e\omega}\phi_{e\alpha}}{\phi_{ee}\phi_{\omega\omega} - (\phi_{e\omega})^2} > 0 \because \phi_{e\omega} > 0, \phi_{e\alpha} > 0, \phi_{\omega\alpha} > 0 \tag{107}$$

Differentiating (104) and (105) w.r.t. α and simplifying we get

$$\begin{aligned} \frac{d\omega^S}{d\alpha} &= \frac{\phi_{\omega\alpha}|\phi_{ee}| + \frac{\beta - \{(1+\gamma)(1-\mu)+\delta\mu\}\theta^F}{2\beta + \{(1+\gamma)(1-\mu)+(1-\delta)\mu\}\theta^S - \{(1+\gamma)(1-\mu)+\delta\mu\}\theta^F}\phi_{e\omega}\phi_{e\alpha}}{\phi_{ee}\phi_{\omega\omega} - (\phi_{e\omega})^2} \\ \frac{d\omega^F}{d\alpha} &= \frac{\phi_{\omega\alpha}|\phi_{ee}| + \frac{\beta - \{(1+\gamma)(1-\mu)+(1-\delta)\mu\}\theta^S}{2\beta + \{(1+\gamma)(1-\mu)+(1-\delta)\mu\}\theta^S - \{(1+\gamma)(1-\mu)+\delta\mu\}\theta^F}\phi_{e\omega}\phi_{e\alpha}}{\phi_{ee}\phi_{\omega\omega} - (\phi_{e\omega})^2} \end{aligned} \tag{108}$$

(i) Differentiate (99) wrt α we get for $i = F, S$

$$\phi_{e\alpha} + \phi_{e\omega}\frac{d\omega}{d\alpha} + \phi_{ee}\frac{de^{i*}}{d\alpha} = 0 \Leftrightarrow \frac{de^{i*}}{d\alpha} = \frac{\phi_{e\alpha} + \phi_{e\omega}\frac{d\omega}{d\alpha}}{|\phi_{ee}|} > 0 \tag{109}$$

Since $\phi_{e\alpha} > 0, \phi_{e\omega} > 0, \frac{d\omega}{d\alpha} > 0 \Rightarrow \frac{de^{i*}}{d\alpha} > 0$. If $\omega = \omega^{FB}$ then $\frac{de^{i*}}{d\alpha}$ since $\frac{d\omega^{FB}}{d\alpha} > 0$.

If $\omega = \omega^S$ or ω^F, then $\frac{d\omega}{d\alpha} <=> 0$. To examine the sign of $\frac{de^{i*}}{d\alpha}$,we examine it when $\frac{d\omega}{d\alpha}$ is most negative. $\frac{d\omega^S}{d\alpha}$ is most negative if $\gamma = 1, \delta = 0, \beta = 1, \theta^F = 1$ and $\theta^S = 0$.

$$\frac{de^{i*}(\omega = \omega^S)}{d\alpha} > \phi_{e\alpha} + \phi_{e\omega}\frac{\phi_{\omega\alpha}|\phi_{ee}| + \left(1 - \frac{1}{2\mu}\right)\phi_{e\omega}\phi_{e\alpha}}{\phi_{ee}\phi_{\omega\omega} - (\phi_{e\omega})^2} \tag{110}$$

Define $\underline{\mu} = \frac{1}{2}\frac{(\phi_{e\omega})^2\phi_{e\alpha}}{\phi_{ee}\phi_{\omega\omega}\phi_{e\alpha} + \phi_{\omega\alpha}|\phi_{ee}|\phi_{e\omega}}$. Since using concavity $(\phi_{e\omega})^2 < \phi_{ee}\phi_{\omega\omega}$, it follows $\frac{(\phi_{e\omega})^2\phi_{e\alpha}}{\phi_{ee}\phi_{\omega\omega}\phi_{e\alpha} + \phi_{\omega\alpha}|\phi_{ee}|\phi_{e\omega}} < 1$. Therefore, $\underline{\mu} < 0.5$. It is easy to see on simplification

$$\mu > \underline{\mu} \Rightarrow \phi_{e\alpha} + \phi_{e\omega}\frac{d\omega^S}{d\alpha} > 0 \Longleftrightarrow \frac{de^{i*}(\omega = \omega^S)}{d\alpha} > 0 \forall \alpha, \delta, \gamma, \beta, \theta^S, \theta^F \tag{111}$$

Similarly, $\frac{d\omega^F}{d\alpha}$ is most negative if $\gamma = 1, \delta = 0, \beta = 1, \theta^F = 0$ and $\theta^S = 1$. Therefore,

$$\mu > \underline{\mu} \Rightarrow \frac{de^{i*}(\omega = \omega^F)}{d\alpha} > 0 \forall \alpha, \delta, \gamma, \beta, \theta^S, \theta^F \tag{112}$$

Next, set $\gamma = 1, \mu = 0, \theta^S = 0, \beta = 1$

$$\frac{de^{i*}(\omega = \omega^S)}{d\alpha} > \phi_{e\alpha} + \phi_{e\omega}\frac{\phi_{\omega\alpha}|\phi_{ee}| + \frac{1-2\theta^F}{2-2\theta^F}\phi_{e\omega}\phi_{e\alpha}}{\phi_{ee}\phi_{\omega\omega} - (\phi_{e\omega})^2} \tag{113}$$

Define $\overline{\theta} = 1 - \frac{1}{2}\frac{(\phi_{e\omega})^2\phi_{e\alpha}}{\phi_{ee}\phi_{\omega\omega}\phi_{e\alpha} + \phi_{\omega\alpha}|\phi_{ee}|\phi_{e\omega}}$.Since using concavity $(\phi_{e\omega})^2 < \phi_{ee}\phi_{\omega\omega}$, it follows $\frac{(\phi_{e\omega})^2\phi_{e\alpha}}{\phi_{ee}\phi_{\omega\omega}\phi_{e\alpha} + \phi_{\omega\alpha}|\phi_{ee}|\phi_{e\omega}} < 1$. Therefore, $\overline{\theta} > 0.5$. It is easy to see on simplification

$$\theta^F < \overline{\theta} \Rightarrow \phi_{e\alpha} + \phi_{e\omega}\frac{d\omega^S}{d\alpha} > 0 \Longleftrightarrow \frac{de^{i*}(\omega = \omega^S)}{d\alpha} > 0 \forall \alpha, \delta, \gamma, \beta, \mu, \theta^S \tag{114}$$

Similarly,

$$\theta^S < \overline{\theta} \Rightarrow \phi_{e\alpha} + \phi_{e\omega}\frac{d\omega^F}{d\alpha} > 0 \Longleftrightarrow \frac{de^{i*}\left(\omega = \omega^F\right)}{d\alpha} > 0 \forall \alpha, \delta, \gamma, \beta, \mu, \theta^F \tag{115}$$

(ii) Differentiating (103) , (104) and (105) w.r.t. δ and simplifying we get

$$\frac{d\omega^{FB}}{d\delta} = \frac{d\omega^S}{d\delta} = \frac{d\omega^F}{d\delta} = 0 \tag{116}$$

Differentiate (99) wrt δ we get for $i = F, S$

$$\begin{aligned} \phi_{ee}\frac{de^{S*}}{d\delta} + \phi_{e\omega}\frac{d\omega}{d\delta} &= \frac{2\mu\theta^S}{\left[\beta + \{(1+\gamma)(1-\mu) + (1-\delta)\mu\}\theta^S\right]^2} \\ \phi_{ee}\frac{de^{F*}}{d\delta} + \phi_{e\omega}\frac{d\omega}{d\delta} &= -\frac{2\mu\theta^F}{\left[\beta + \{(1+\gamma)(1-\mu) + \delta\mu\}\theta^F\right]^2} \end{aligned} \tag{117}$$

Using (116) it follows

$$\begin{aligned} \frac{de^{S*}}{d\delta} &= -\frac{2\mu\theta^S}{\left[\beta + \{(1+\gamma)(1-\mu) + (1-\delta)\mu\}\theta^S\right]^2} < 0 \Leftrightarrow e^{S*}(\delta = 1) < e^{S*}(\delta = 0) \\ \frac{de^{F*}}{d\delta} &= \frac{2\mu\theta^F}{\left[\beta + \{(1+\gamma)(1-\mu) + \delta\mu\}\theta^F\right]^2} > 0 \Leftrightarrow e^{F*}(\delta = 1) > e^{F*}(\delta = 0) \end{aligned} \tag{118}$$

which completes the proof.$\diamondsuit$

Proof of Proposition 16: Note that, because the scientist has all the bargaining power at date 0 and the financier is not liquidity constrained, the participation constraint (27) must be binding in the optimal contract, which implies that

$$q^*(s) = U - \Pi^{S*} \tag{119}$$

so that

$$\Pi^{F*} + s - q^*(s) = \Pi^{F*} + \Pi^{S*} + s - U \tag{120}$$

$$\underset{(\alpha,\delta,\Omega),w(s)}{\arg\max} \left[\Pi^{F*} + s - q^*(s)\right] \equiv \underset{(\alpha,\delta,\Omega)}{\arg\max} \left[\Pi^{S*} + \Pi^{F*} + s\right] \tag{121}$$

which completes the proof.$\diamondsuit$

Proof of Proposition 16:

$$p^S = 0.5\left[R\left(0, e^{FF}\right) + r^S\left(0, e^{FF}\right) - r^F\left(0, e^{FF}\right)\right] \tag{122}$$

$$p^F = 0.5\left[R\left(e^{SF}, 0\right) + r^F\left(e^{SF}, 0\right) - r^F\left(e^{SF}, 0\right)\right] \tag{123}$$

Suppose $e^{S*} \neq e^{SF}$. If the Financier exercises his option to sell (if selected) and the scientist finds it individually rational to not leave the relationsip, then the financier's payoff is

$$\begin{aligned} & p^S + 0.5\left[R\left(e^S, e^F\right) + r^S\left(e^S, e^F, J\right) - r^F\left(e^S, e^F, J\right)\right] \\ = \;& 0.5\left[R\left(e^S, e^F\right) + R\left(0, e^{FF}\right) + r^S\left(0, e^{FF}, J\right) + r^S\left(e^S, e^F, J\right) - r^F\left(e^S, e^F, J\right) - r^F\left(0, e^{FF}, J\right)\right] \end{aligned} \tag{124}$$

Clearly, the option to sell will not induce the scientist's investments to be first best since the effect of the scientist's investment on her and the financier's outside option functions do not drop out of the resulting payoff. In fact, even if the exercise prices are made a function of any fixed level of investment, the first-best level of investment by the scientist will not result. A similar argument applies for the financier's investment as well. $\diamondsuit$

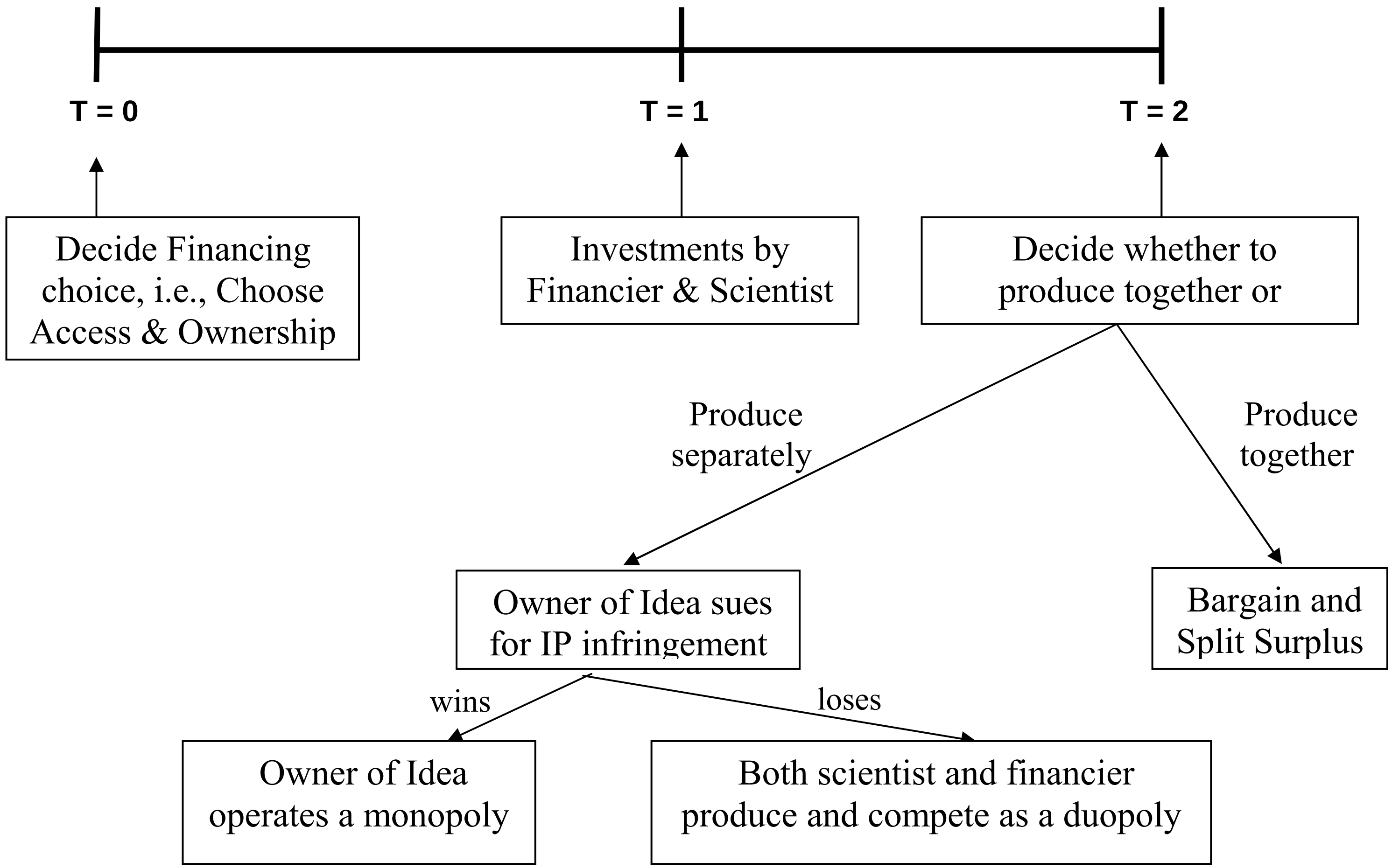

Figure 1: Timing and Sequence of Events

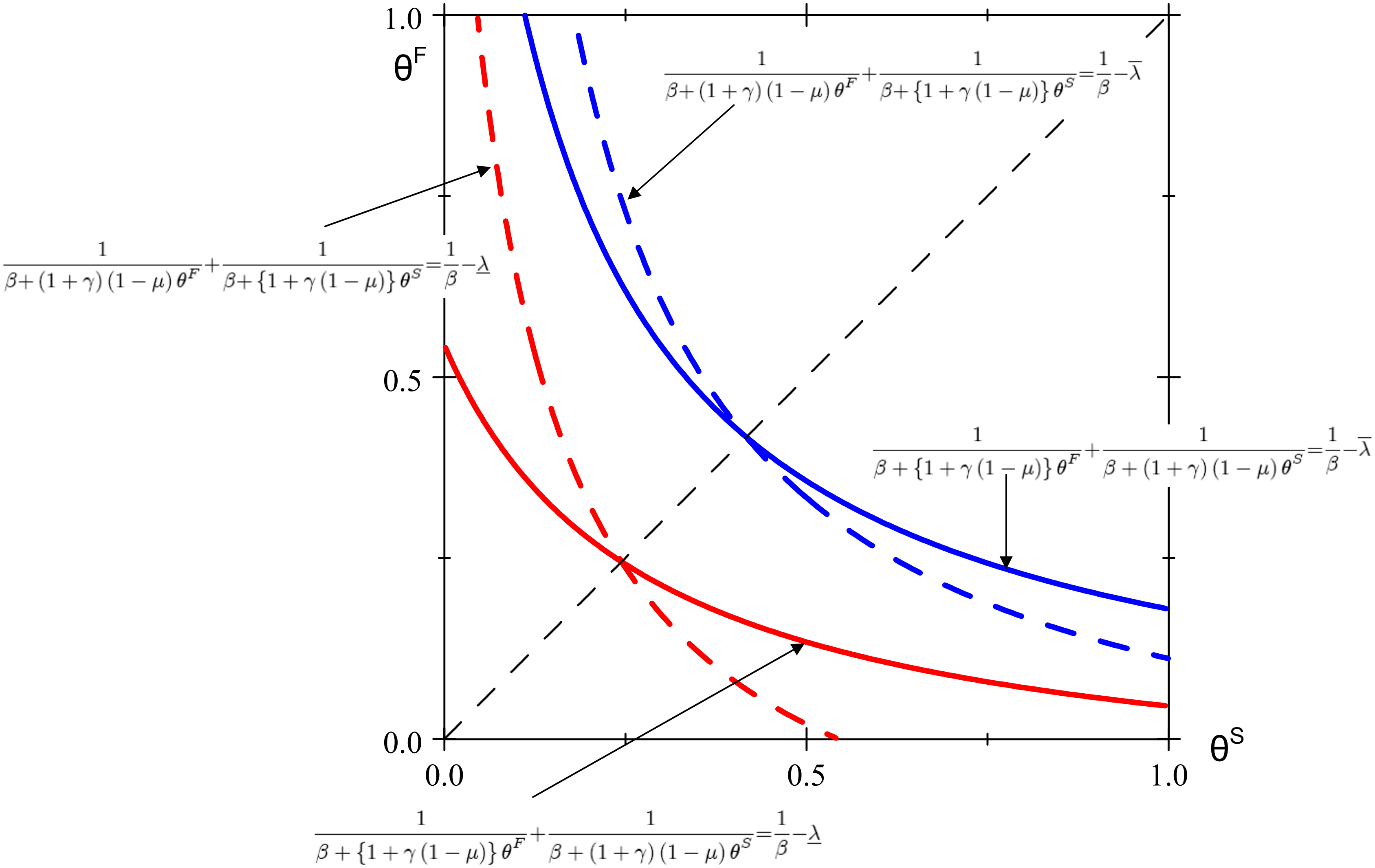

Figure 2: Boundaries deciding the optimal level of knowledge sharing as a function of ease of expropriation

This diagram plots the boundaries specified in Proposition 12 to decide the optimal level of knowledge sharing as as a function of the scientist's and financier's abilities to expropriate knowledge assets – θ^S and θ^F respectively.

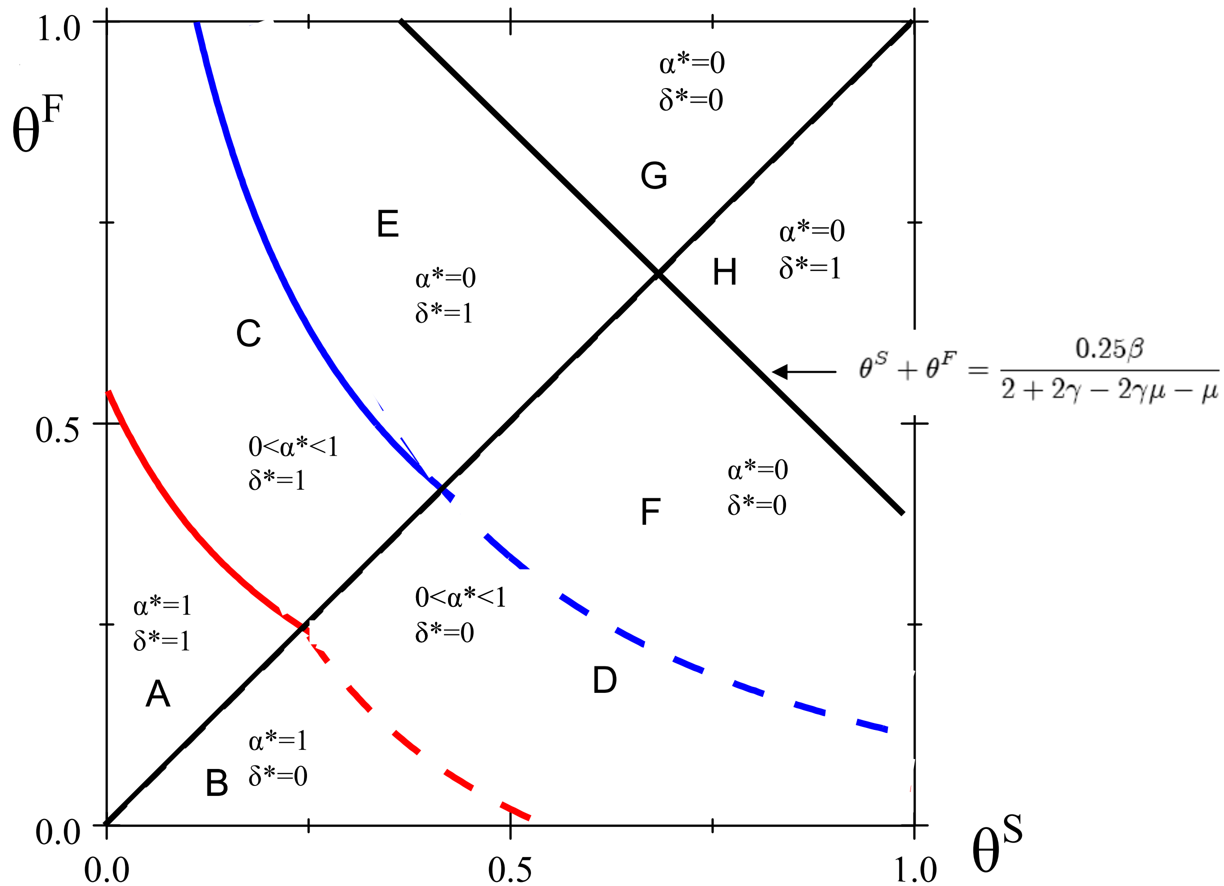

Figure 3: Optimal financing choice as a function of ease of expropriation

This diagram plots the optimal financing choice, i.e. optimal knowledge sharing and ownership of the idea, as a function of the scientist's and financier's abilities to expropriate knowledge assets – θ^S and θ^F respectively. $\alpha^*=0$, $0<\alpha^*<1$ and $\alpha^*=1$ denote minimal, moderate and full knowledge sharing respectively. $\delta^*=0$ and $\delta^*=1$ respectively denote the scientist and the financier owning the idea. The top (bottom) set of lines display the boundary between zero (full) access and interior solutions for access.